Paper

Input Effects and Sensory Receptor Responses of Triangular Oscillator Networks

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Synopsis: Properties of 1–D oscillators for fixed inputs show that finite range activities are limited to the input level. Thus the input ranges of the 1–D oscillator are defined. Triangular networks organized with three 1–D oscillators give singular outputs to same level rectangular inputs. The responses to sets of differing input level combinations are summarized in a relation diagram.

Keywords: receptive field; one-dimensional oscillator; network.

1. Introduction

We have been studying the nature of 1–D oscillator networks [2–4]. We designed two types of 1–D oscillators [1] that are analogs of nerve cells that have two different features, namely excitatory and inhibitory responses that we refer to as P– and N–type oscillators. Thus, networks consisting of these two kinds of elements will be expected to have multiple functions that reveal changes in the dynamical behaviors of the network. In other words, any resulting response functions are due to the organized sequence of dynamics in these systems.

We therefore decided to consider sensory receptive field in terms of the organized behavior of a system of collaborating P and N elements in response to external inputs that applied to the system. Usually, the term *receptive field* refers to an N–dimensional portion (field) of sensory space over which biological systems are sensitive to the surrounding world. However the responses of receptive fields to the inputs derived from surrounding environmental circumstances can be regarded as the whole systematic dynamics organized from the dynamics of each element in the sensory receptive system. Receptive fields therefore become a more general concept when one considers them in terms of the behavior of organized systems of any type. Our understanding of a receptive field is thus in terms of the response characteristics to the input features. In this sense, it becomes a subject for consideration as to what kind of receptive field will be found in the 1–D oscillator networks that have been studied [1, 2] by us. The present paper is devoted to investigating the properties of elemental 1–D oscillators with respect to inputs and response features of a triangle network, each oscillator of which receives its own input from a single receptor. The response properties of the network to inputs are thus realized in the light of the nature of the elemental oscillators. A study consider-

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ing the response dynamics for inputs to the isolated 1–D oscillators is preparatory to understanding the dynamics of the network response. Networks consisting of three 1–D oscillators, so called *triangle networks*, are simple and fundamental to investigating the properties of networks. We therefore start the study of the dynamical behavior of triangular networks to inputs as a step toward understanding the concept of the receptive field of general 1–D oscillator networks.

2. Input Effects and Input Range

2.1. Input Effects

Generally, input effects of an oscillatory element are determined by how we modify the function that describes the temporal developments of the element. Let an oscillatory element be described by the following recurrence equation with a parameter A,

$$x_{t+1}=f(x_t;\ A). \tag{2.1}$$

When an input $I(t)$ is applied to the 1–D oscillator, we can imagine the following four modifications of the recurrence equation (2.1),

I: Changes in the x_{t+1} direction

(a) Shift $x_{t+1}=f(x_t;\ A)+\sigma I(t)$ (2.2.a)
(b) Zoom $x_{t+1}=\sigma I(t)\cdot f(x_t;\ A)$ (2.2.b)

II: Changes in the x_t direction

(a) Shift $x_{t+1}=f(x_t+\sigma I(t);\ A)$ (2.3.a)
(b) Zoom $x_{t+1}=f(\sigma I(t)\cdot x_t;\ A)$ (2.3.b)

where σ signifies a connecting weight for the input. Basically these modifications of the function $f(x_t;\ A)$ encompass addition and multiplication. Addition implies upward or downward shifts, and the multiplication implies zooming of the function. The modifications encompass two directions, namely, along the x_{t+1} axis (case I) or along the x_t axis (case II). The above four modifications of the recurrence equation are illustrated in Fig. 1. The usual input is taken in the form (2.2.a), i.e., case I–(a) (dashed curve). As shown in Fig. 1, this type of input modifies the mapping only by shifting the original map (the function $f(x_t;\ A)$) up or down. In case II where the input combines with the x_t value, the mapping of the recurrence equation keeps its original form but changes the x_t-variable region to shift and to zoom along the x_t direction.

2.2. Input Range

The 1–D oscillator used follows our previous work [1–3]. The 1–D oscillator is described with a recurrence equation of unit interval [0, 1] onto the same interval using a cubic function with a parameter A, moving over the range [0, 4]. The interesting range is [1, 4]. To adapt 1–D oscillators to receptive field modeling where input effects will be considered, a particular 1–D oscillator element is modified to have the additional input term I(t), namely

$$x(t+1)=f(x_t;\ A)+\sigma I(t)=\pm Ax(t)\ (1-x^2(t))\pm x(t)+I(t). \tag{2.4}$$

Note that we set that the connecting weight σ to be 1.

Since the input term $I(t)$ shifts the original cubic-function up or down, the recurrence equation (2.4) generates a temporally diverging series of the state variable in some area of the parameter A with respect to the region of initial value $x(0)$. We therefore define the *input range* to be where the recurrence equation (2.4) generates finite values within an infinite time. The cubic function used here is rotationally symmetric with respect to the origin (0, 0) so that the input range is defined for the interval $[-I_r, I_r]$ as illustrated in Fig. 2. The actual input ranges of the 1–D oscillators are shown in Figs 3 to 5. The input range for initial values is shown in Fig. 3 where the horizontal line denotes that of the N–type oscillator and curved line the P–type. The input ranges for the parameter A are also depicted in Fig. 4 for the P–

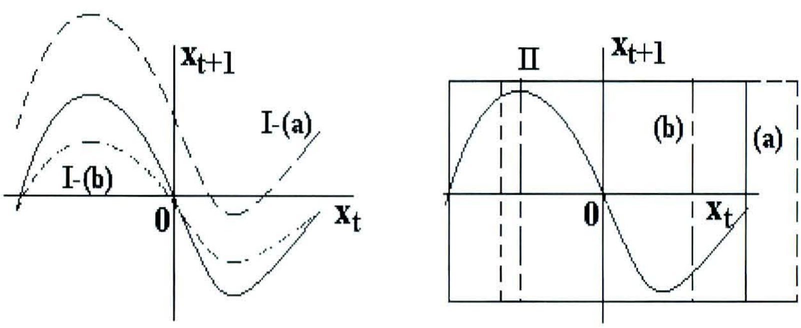

Fig. 1 Modified maps illustrating the four types of input effects.

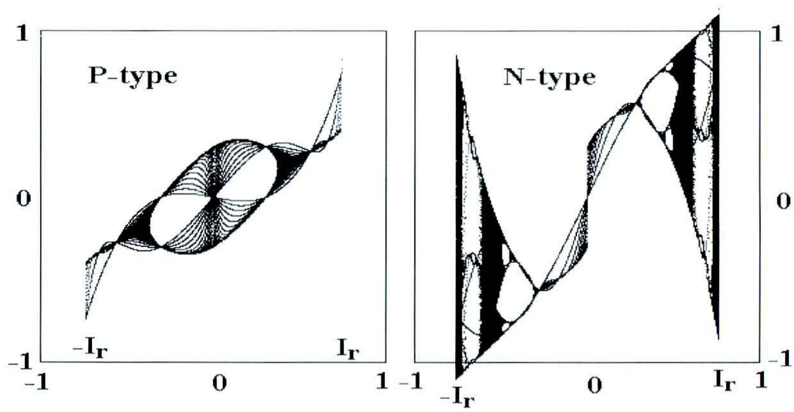

Fig. 2 Actual demonstration of the working area of a P type 1–D oscillator (left side) and a N type1–D oscillator (right side). The input range is denoted by $\pm I_r$.

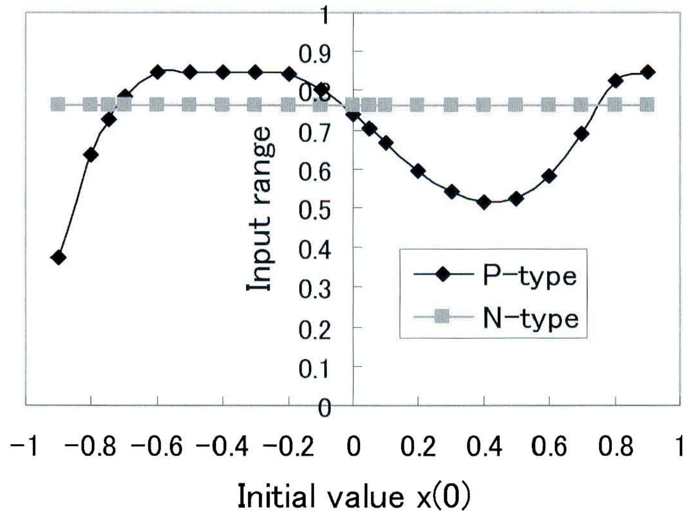

Fig. 3 Input range for initial value x(0) of both oscillators. The horizontal line signifies the input range of N–type oscillator while the curved line is that of P–type oscillator. The value of the parameter A is 2.2.

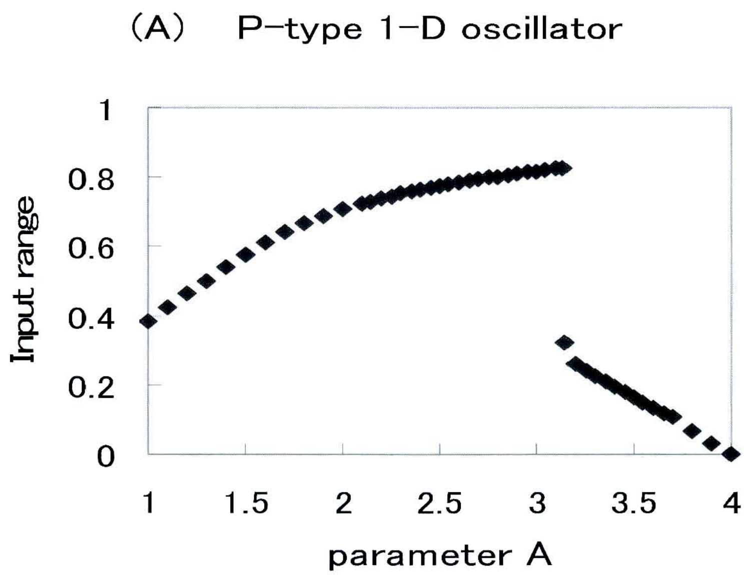

Fig. 4 Input ranges of P–type oscillator for the parameter A. The initial value x(0) is 0.2

type oscillator and Fig. 5 for the N–type oscillator. There can be seen discontinuous jump within the input range curves. The P–type oscillator has a big discontinuity near A=3.1. We also see that the N–type oscillator can work outside the unit interval [−1,1] between A=1 and A=1.5. The input range of N–type oscillator gradually decreases from A=1.5 to A=4. These points are different for the input range of P–type oscillator.

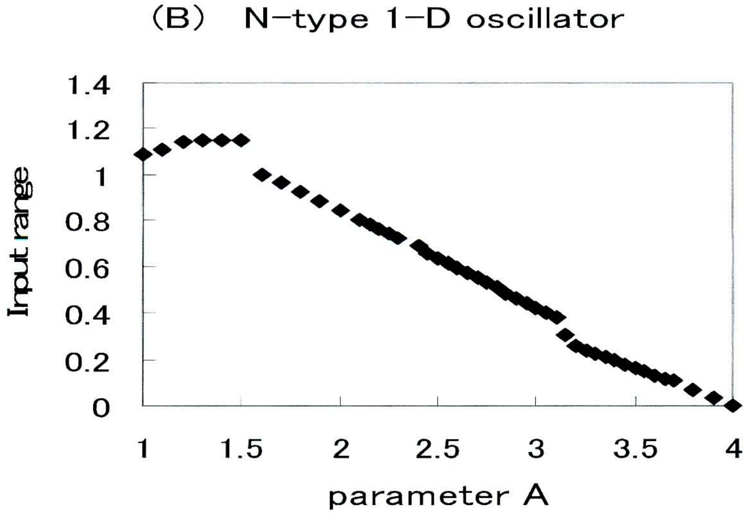

Fig. 5 Input range of N-type oscillators for the parameter A. Initial values as in Fig. 4.

These input range investigations suggest that working range of 1-D oscillators operating in a network would be narrower. We can easily collapse the numerical calculations when the number of connections between oscillators becomes large or the oscillator parameter A is large. Numerical calculations will be meaningful when the input height is kept to low levels and the parameter A is restricted in the lower range.

3. Response properties of a triangle network to rectangular inputs

This section is devoted to the responses of the triangular network to temporally rectangular inputs of duration 20 iteration steps. The structure of the triangular network considered here is illustrated in Fig. 6. The network consists of three 1-D oscillators each of which receives its input from an independent receptor. The temporal development of each 1-D oscillator is governed by the following recurrence equation,

$$x_j(t+1) = \pm A_j(1-x_j^2(t))x_j(t) \pm x_j(t) \pm \frac{1}{2}\sum_{k=1}^{3} x_k(t)(1-\delta_{k,j}) \pm R_j(t), \quad j=1,2,3 \tag{3.1}$$

where $\delta_{j,k}$ signifies Kronecker delta, R_j(t) is the input from receptor j.

The triangular networks investigated have four possible configurations of P-type and N-type oscillators, namely an all P-type oscillator network denoted by {P, P, P}, an all N-type oscillator network {N, N, N}, two P-type oscillators and one N-type oscillator network {P, P, N}, and one P-type oscillator and two N-type oscillators {P, N, N}. Figures 7 to10 show the numerical results from those four configurations of oscillators. The {P, P, P} network (Fig. 7) and the {N, N, N} network (Fig. 8) responded with a state development that just followed the input shape. The {P, P, N} network (Fig. 9) and the {P, N, N} network (Fig. 10)

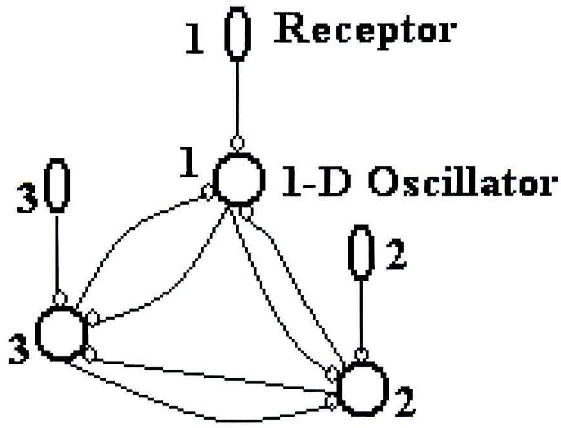

Fig. 6 Scheme of a triangle oscillator network. The circles are 1–D oscillator elements and the ovals are putative sensory receptor mechanisms.

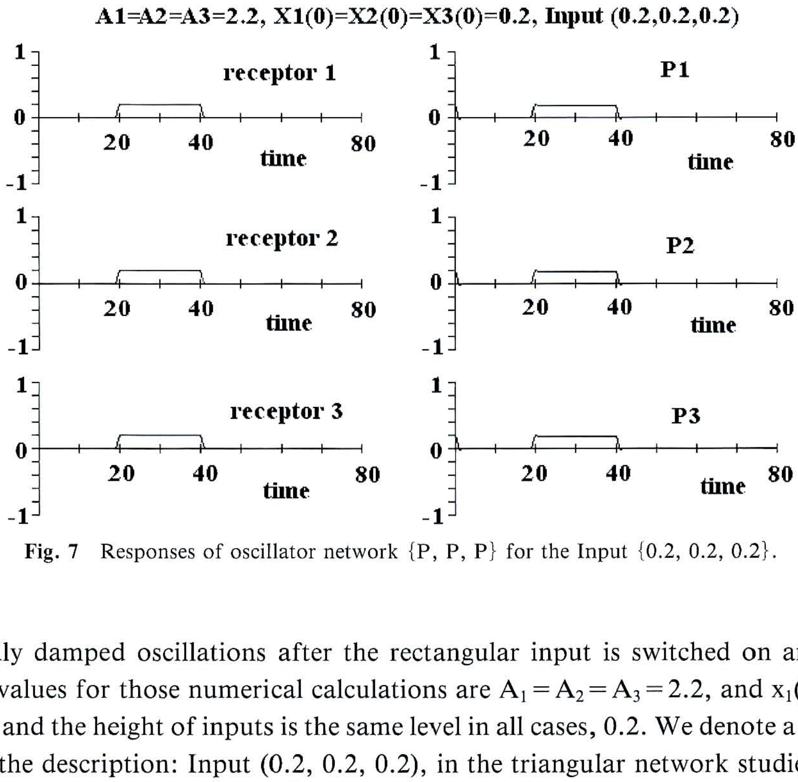

Fig. 7 Responses of oscillator network {P, P, P} for the Input {0.2, 0.2, 0.2}.

show rapidly damped oscillations after the rectangular input is switched on and off. The parameter values for those numerical calculations are $A_1 = A_2 = A_3 = 2.2$, and $x_1(0) = x_2(0) = x_3(0) = 0.2$, and the height of inputs is the same level in all cases, 0.2. We denote a set of input heights by the description: Input (0.2, 0.2, 0.2), in the triangular network studied here, the fractional numbers in the Input (0.2, 0.2, 0.2) referring to the input levels each of the three oscillators, respectively.

The numerical results for the Input (0.2, 0.2, 0.2) case is special since the input levels of all oscillators are the same. To see this fact we made one of inputs slightly different from the

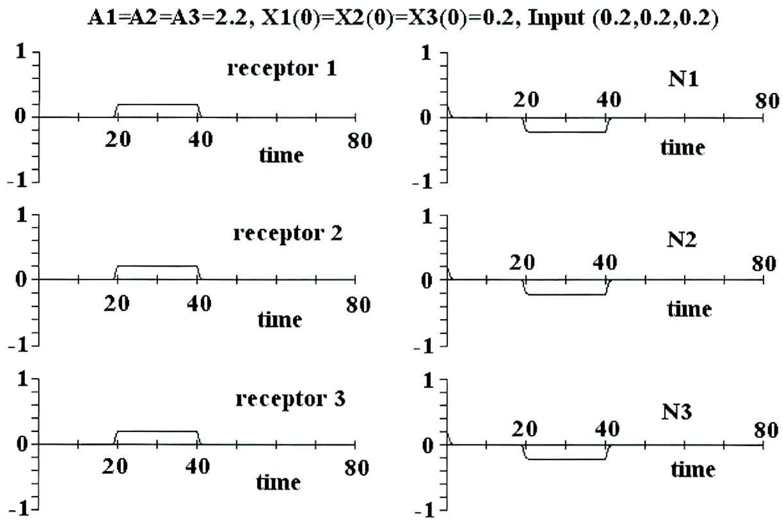

Fig. 8 Responses of oscillator network {N, N, N} for the Input {0.2, 0.2, 0.2}.

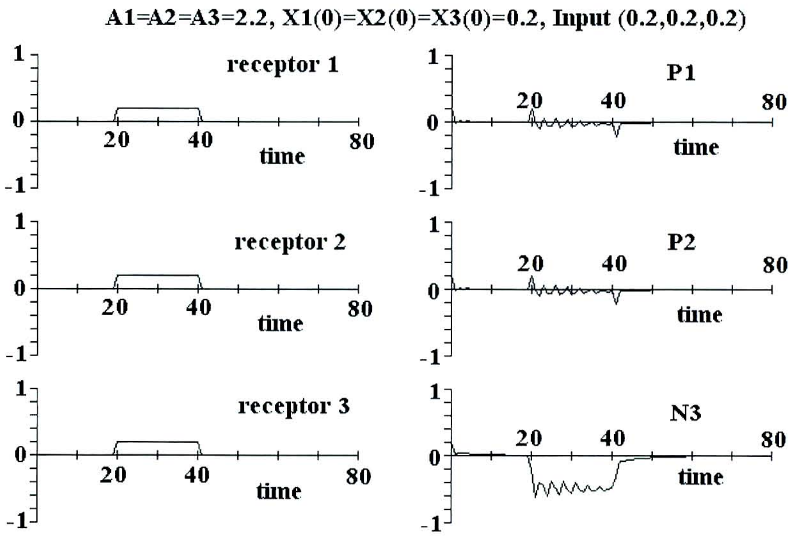

Fig. 9 Responses of oscillator network {P, P, N} for the Input {0.2, 0.2, 0.2}.

other two. The results of numerical calculations for the Input (0.2, 0.2, 0.19) are shown in Figs 11 to 14. The responses are different from those to Input the (0.2, 0.2, 0.2) in the networks {P, P, P}, {N, N, N} and {P, N, N}, while the responses are the same as those of the Input (0.2, 0.2, 0.2) for {P, P, N} network. It is obvious that Fig. 13 is equivalent to Fig. 9. Thus we know that there exist equivalent responses that depend upon particular different combinations of input levels. The equivalence of the outcomes in Figs 9 and 13 is due to the

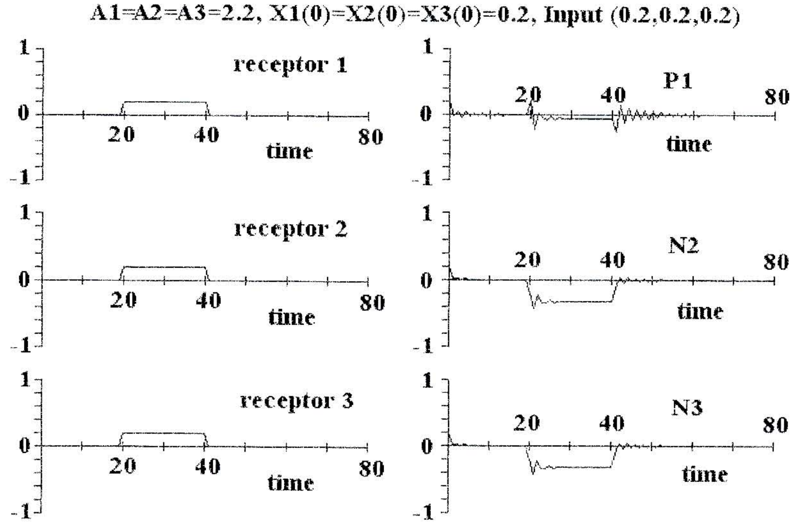

Fig. 10 Responses of oscillator network {P, N, N} for the Input {0.2, 0.2, 0.2}.

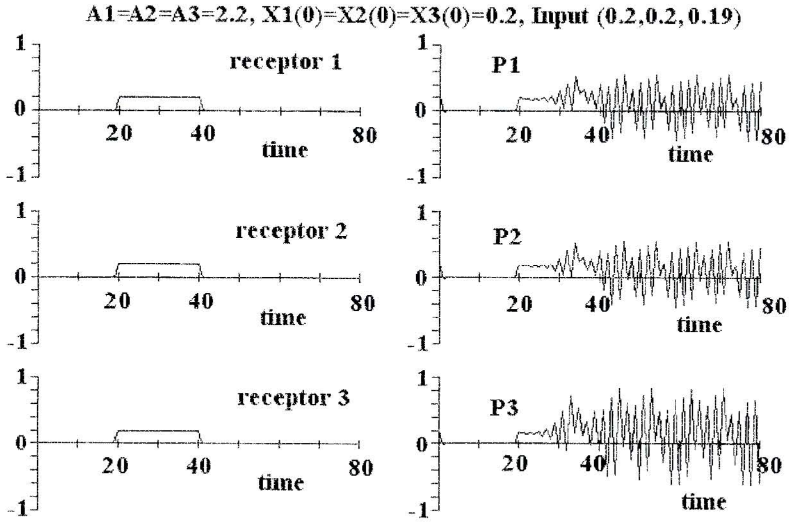

Fig. 11 Responses of oscillator network {P, P, P} for the Input {0.2, 0.2, 0.19}.

two P-type oscillators having the same inputs. This can be seen from the fact that the numerical results for the network {P, P, N} for the Input (0.2, 0.19, 0.2) (Fig. 15) are different from those of the Input (0.2, 0.2, 0.2) (Fig. 9) and those of the Input (0.2, 0.2, 0.19) (Fig. 13). In Fig. 16 we summarize the relationships between responses to the combinations of levels of rectangular inputs. The numbers 1, 2, 3, 4 appearing in Fig 16 refer to the networks 1: {P, P, P}, 2: {N, N, N}, 3: {P, P, N}, and 4: {P, N, N}.

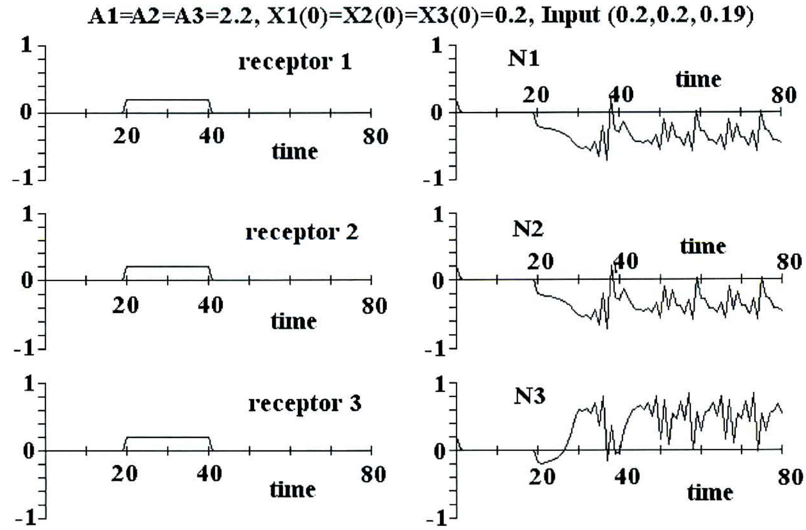

Fig. 12 Responses of oscillator network {N, N, N} for the Input {0.2, 0.2, 0.19}.

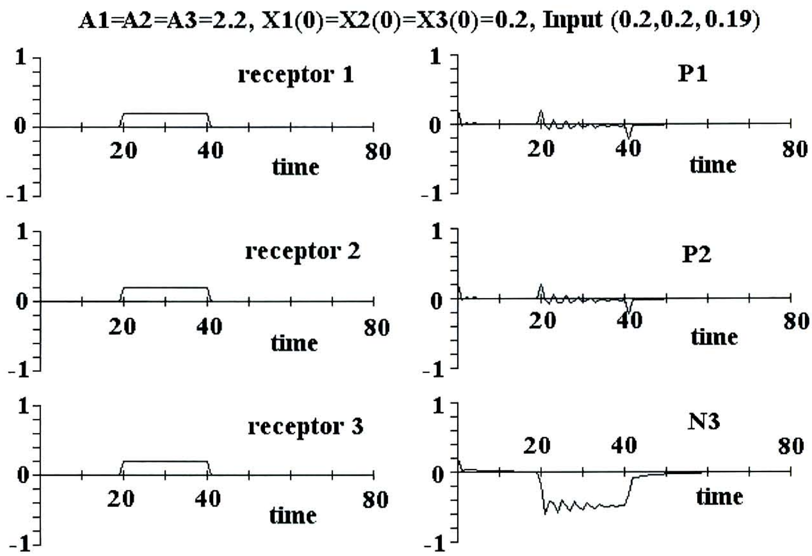

Fig. 13 Responses of oscillator network {P, P, N} for the Input {0.2, 0.2, 0.19}.

The numerical calculations discussed above were performed using the same parameter values and the same initial values, namely, $A_1 = A_2 = A_3 = 2.2$ and $x_1(0) = x_2(0) = x_3(0) = 0.2$. In an isolated 1–D oscillator the parameter value $A = 2.2$ indicates a period 2 oscillation in P–type oscillators and steady state in N–type oscillators. This nature is reflected in the same input level cases, but in almost all cases infinitesimal deviations cause chaotic responses as shown Figs 11, 12, 14, and 15. The chaotic responses continue after inputs are switched off.

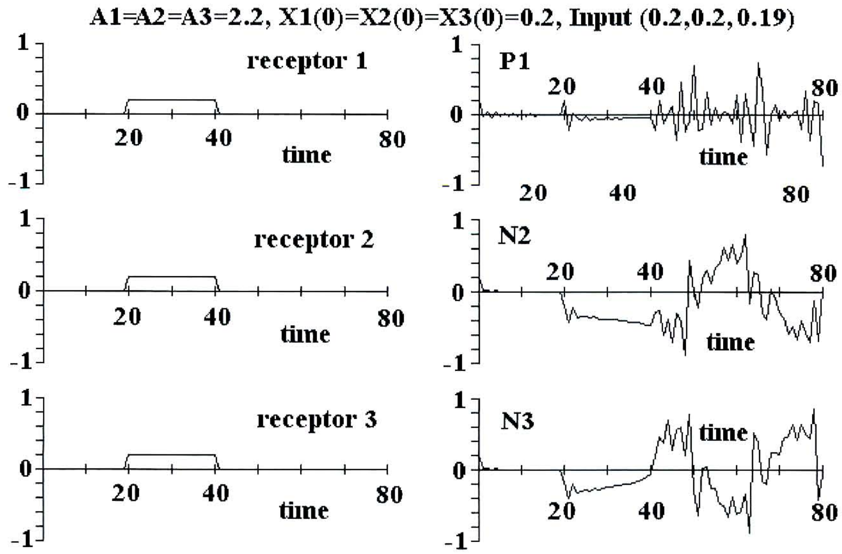

Fig. 14 Responses of oscillator network {P, N, N} for the Input {0.2, 0.2, 0.19}

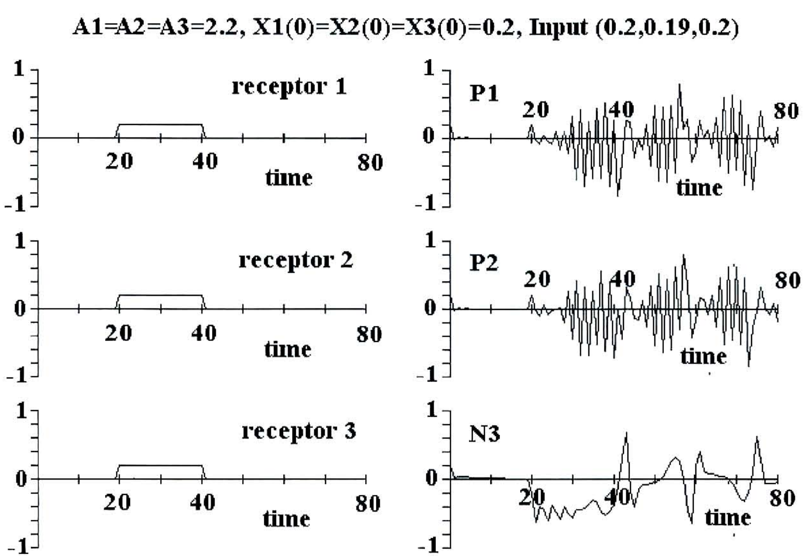

Fig. 15 Responses of oscillator network {P, P, N} for the Input {0.2, 0.19, 0.2}.

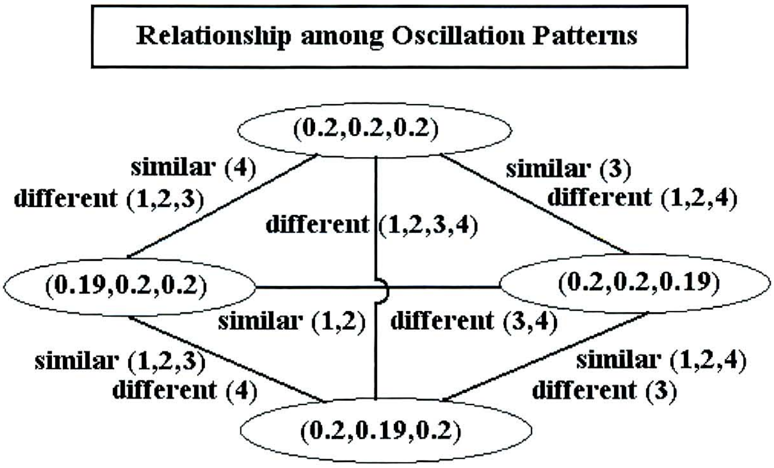

Fig. 16 Relationships among oscillation modes of the four types of networks, 1: {P, P, P}, 2: {N, N, N}, 3: {P, P, N}, and 4: {P, N, N}.

4. Discussion

The responses investigated in the present paper are restricted to within a special parameter space, that is all the parameters (A_1, A_2, A_3) of the oscillators are set to the same value. This case reveals a singular property when all the inputs are of the same level. In this case the recurrence equation (3.1) gives the following recurrence equation

$$Z(t+1)=A(1-Z^2(t))Z(t)+Z(t)+G_{123}(t)Z(t)+U_{123}(t)-Q_{123}(\mathrm{t})+\mathrm{R}(\mathrm{t}) \qquad (4.1)$$

for the additive quantity $Z(t)$ of $x_1(t)$, $x_2(t)$, and $x_3(t)$ with ± 1 weights of k_1, k_2, and k_3, i.e.,

$$Z(t)=k_1x_1(t)+k_2x_2(t)+k_3x_3(t), \qquad (4.2)$$

where $G(t)$, $Q(t)$, and $R(t)$ are respectively defined as

$$G_{123}(t)=3A(k_1k_2x_1(t)x_2(t)+k_2k_3x_2(t)x_3(t)+k_3k_1x_3(t)x_1(t)), \qquad (4.3)$$

$$U_{123}(t)=\frac{k_2+k_3}{2}x_1(\mathrm{t})+\frac{k_3+k_1}{2}x_2(\mathrm{t})+\frac{k_1+k_2}{2}x_3(t) \qquad (4.4)$$

$$Q_{123}(t)=3Ak_1k_2k_3x_1(t)x_2(t)x_3(t), \qquad (4.5)$$

and

$$R(t)=k_1R_1(t)+k_2R_2(\mathrm{t})+k_3R_3(t). \qquad (4.6)$$

Note again that $k_1=\pm 1$, $k_2=\pm 1$, and $k_3=\pm 1$.

The additive quantity $Z(t)$ is useful for considering the network responses analytically. If we consider networks consisting of all the same types of oscillators ($k_1=k_2=k_3=k$), the following relations appear,

$$U_{123}(t)=Z(t), \tag{4.7}$$

and

$$G_{123}(t)=\frac{3}{2}A(Z^2(t)-k^2(x_1^2(t)+x_2^2(t)+x_3^2(t))). \tag{4.8}$$

These relations reduce the recurrence equation (3.1) to the following difference equation,

$$Z(t+1)=A\left(1+\frac{1}{2}Z^2(t)\right)Z(t)+(2-3(x_1^2(t)+x_2^2(t)+x_3^2(t)))Z(t)-Q_{123}(t)+R(t). \tag{4.9}$$

The recurrence equation (4.9) keeps its form if we take $Z(t)$ to be the average of three oscillator variables instead of the additive quantity (eq. (4.2)). If the averaged $Z(t)$ are taken, the quantity $Z(t)$ becomes a variable within the interval $[-1, 1]$. For the averaged $Z(t)$ case it is easily known that the first term $A\ (1+Z^2(t)/2)\ Z(t)$ reveals that the origin is a source, in other words, the origin is unstable so that $Z(t)$ diverges. The first term implies that the responses of the triangular network easily generate chaotic time series or divergent time series. Moreover the first term lets the working area of the network be narrow.

When considering the role of the second and third terms in eq. (4.9) it becomes clear that the second term becomes important since the first term causes instabilities. The third term $Q_{123}(t)$ takes positive or negative values depending on the variables $x_1(t)$, $x_2(t)$, and $x_3(t)$. The second term has a negative gradient when the following inequality is satisfied

$$(x_1^2(t)+x_2^2(t)+x_3^2(t))>\frac{2}{3}.$$

A rough estimation of the x variable value that gives a negative gradient for the second term is $x(\mathrm{t})>\sqrt{2}/3\approx 0.471$, if it is assumed that all x variables take the same value. The negative gradient of the second term can produce the stabilization of the recurrence equation (4.9) if the absolute value of the gradient of this term is larger than the gradient of the first term. If stabilization occurs, the origin becomes a sink point. A sink origin can let the recurrence equation (4.9) be $Z(t+1)=R(t)$. However, the actual mechanism to realise the responses shown in Figs 7 to10 is more complicated than the mechanism discussed above.

In the cases of different values of A, the above method is also applicable if each parameter A_j is divided into average and deviation terms, i.e., $A_j=\langle A\rangle+\Delta A_j$. This case yields additive terms to the eq. (4.1). We therefore understand that the perturbation method is still applicable if the additive terms are small enough, but the system of equation generates entirely different temporal developments if the additive terms carry strong instabilities.

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