Paper

Mechanics of Control Dynamics

YOSHINORI NAGAI* and TED MADDESS**

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Synopsis: Control is considered from the viewpoint of dynamics. Causality between dynamics is required to establish the control. Thus control can be discussed as a dynamical behavior of a system. A mathematical description for such control is presented. Numerical results using one-dimensional oscillator maps are shown to demonstrate that the concepts considered here can provide effective control. The control revealed from dynamics is called control dynamics.

Key words: control dynamics, causality, memory effect, oscillator map

1. Introduction

In this article, we consider the notion of control between dynamics and system. Usually, "control" means an aimed sequence of events with causal relationship. When a mechanism maintains a designed function in an automatic way, we say it is controlled. Steady or cyclic development of a system appears to be controlled dynamics. These thoughts lead to a concept for control that coupled dynamics can be organized to form the regulation of a system. Thus, we do not require any *a priori* matters for considering control. Instead we consider control based on ordinary concepts of dynamics. Our starting point for discussing control is that any control is a class of dynamics. As anyone intuitively knows, there exist at least two kinds of dynamics in the control so that the control is established in the dynamics of many component systems. Generally each component forming a control dynamics also has many elementary dynamics of dynamics, like particles. In other words, controls appear in many-body systems.

Dynamics [1] describes the nature of existence in the situation where all the existence stands on substantial materials. Control is organized by interactive dynamics. As is well known, a control forms a lord and vassal relationship in dynamics. Elements of a system are distributed in a space with their interactions. Even if each element of system has the same property to each other, a group of gathering elements in a local area can generate a different dynamical behavior in another area of the system. Differences are produced brought by phase transitions [2] of local areas. This is the elementary process for formation of a control system. If a system consists of homogenously distributed elements all of which have the same properties, some spontaneous domain transformation will occur and then a domain will per-

* Center for Information Science and Faculty of Political Science & Economics, Kokushikan University.
** Center for Visual Sciences, Research School of Biological Sciences, Australian National University.

form its own dynamics that are restricted by the surrounding domains [3,4,5]. As a result the system creates complicated dynamics. For the complicated dynamics created to organize a controlled system, it is required that some parts of the complicated dynamics are an obviously repeated feature that the condition for dynamics is the same.

We can apply the above idea for control dynamics to intelligent systems and/or biological systems. Spontaneously appearing dynamics in a domain should have a several parameters (at least one parameter) that bring about the controlling behavior of dynamics when the parameters are changed temporally. These parameters are generated by domains forming elements, even if the original dynamics has no parameters. For example, the boundary of a domain becomes a parameter since some different boundary conditions can cause new dynamics in the domain. As known from such considerations, the dynamics contribute to controls are macroscopic ones organized by microscopic elemental interactions. Thus, we know that the control system consists of inhomogeneous elements that realize macroscopic or mesoscopic [6[level dynamics with parameterization. The control dynamics also require reproducibility. Such reproducibility is related to memory effects.

A memory of an event consists of three processes, namely, making the memory, keeping the memory, and reproducing the memory as summarized by Hebb [7]. The process of making memory may be realized by utilizing a basin for initial values in a dynamical system, or by coupling effects of elements that cause a change of dynamics through the interaction of elements. The maintenance of memory is often hidden when we consider what memory is, but for a dynamics viewpoint of memory the process of keeping memory implies that the dynamics concerning the memory are perpetually maintained eliminating any decay of the organized dynamics, keeping the parameters in the making memory stage. In this sense, there exists a memory maintenance process as a local area dynamics of whole dynamics of the system. The process to keep memory, however, is buried in the dynamics of the entire system. So it is difficult to find the dynamics corresponding to the memory maintenance process in any real system.

The process of reproducing memory is encapsulated by control dynamics. We guess that reproducing memory is a one way control from control dynamics to the memory dynamics. The process is realized by interactions between the control system and the memory system. It is possible to feedback from the memory part to the control part in the system, but the interaction may have different strengths in each part. A possible process to reproduce memory is for the control dynamics to change the memory dynamics through interactions.

The point of memory interest in any processes is the causality between the control dynamics and the memory dynamics. When the dynamics B always appears in memory part following dynamics A in control part, we say the dynamics B is reproduced by the dynamics A. Thus, the generalization leads to the notion that memory effects are general in control dynamics. This viewpoint asserts that memory resides in the causality relationships between dynamics. Thus, we can discard maintenance process of dynamics for keeping the memory. The only requirement is that causality should be kept if a relation between the dynamics forms a memory.

In the next section, we present our concepts for control dynamics and a mathematical description of control dynamics based upon a set of first order differential equations. We also show how control dynamics are derived from a whole system by dividing the whole into subsystems in nested sections. In section 3, we demonstrate that the materials discussed in section 2 are realizable using a one dimensional oscillating map, that will be simply referred oscillator. At first we will study the properties of the one dimensional oscillating map introduced here, and then organize a three oscillators system to investigate the behavior of changes introduced by interactions and coupling. Two kinds of interactions are considered to compare the dependence of the interactions. The interactions studied are the average and triple product of the three oscillator variables. Section 4 is devoted to summary and discussions.

2. Mathematical description of control dynamics

As mentioned in the introduction, the causality relationship is an important point in order to think about what control is. Hence paired parts of two dynamics form a control relationship when parts of the dynamics organize a causal relationship. It is also required that the local development of dynamical variables is recognizably different from the other part of temporal development of variables for dynamics. Thus, we expect that there will exist pre-control behavior of the dynamics. A control will be formed through the interaction between the dynamical systems or variables. The property of the control is determined by the nature of interaction.

We state the requirements for what control is first, and then describe the control mathematically. We present a mathematical view of the scope for what kinds of interactions realize control from dynamics.

2.1 Requirements for control dynamic

As discussed in the introduction, we require the following three attributes for what dynamics reveal control.

I. Existence of several dynamics and interactions among them.

II. Heterogeneous organization or separation of dynamics.

III. Causality between different dynamics.

Since control is a relationship wherein one system governs another, subsystem structure is required. The subsystems will develop temporally so the dynamics must exist there. Point I above indicates this situation. If the subsystem forms a control relationship, fairly equal dynamics are impossible. Thus Point II is established. Control always predicts the same result if the system is controlled. This implies the reproducibility of dynamics for particular boundary conditions or parameter values. Thus, controlled dynamics will always appear after the controlling dynamics has occurred. This is the ordinary concept for control. Hence, Point III is required.

2.2 Mathematical description

A dynamical system can be described by a set of first order differential equations [1], namely,

$$\frac{d}{dt}x = f(x), \tag{1}$$

where x denotes a vector of variables to describe the dynamics of a system. If the system consists of two dynamic elements denoted by the two vectors x_1 and x_2, the interacting dynamics are described as follows:

$$\frac{d}{dt}x_1 = f_1(x_1) + \eta_1(x_1, x_2), \tag{3}$$

$$\frac{d}{dt}x_2 = f_2(x_2) + \eta_2(x_1, x_2). \tag{4}$$

If the dynamics x_1 controls the dynamics x_2, it is expected that $\eta_1(x_1, x_2) \approx 0$. This case provides one-way control. When the solution of the differential equation $(d/dt)x_1 = f_1(x_1)$ is $x_1 = G_1(t)$, one-way control of the dynamics reduces to the differential equation

$$\frac{d}{dt}x_2 = f_2(x_2) + \eta_2(G_1(t), x_2). \tag{5}$$

In the case of no coupling between $G_1(t)$ and x_2, the solution of eq. (5) becomes the addition of the integral of $G_1(t)$ to the solution of $(d/dt)x_2 = f_2(x_2)$, namely,

$$x_2(t) = \int f_2(x_2(t))\, dt + \int G_1(t)\, dt + c. \tag{6}$$

Here x_2 is explicitly written as $x_2(t)$ with time dependence. It is difficult to say there is a control system from eq. (6), when the former part is dominant in the solution. However, when the former term is small enough the solution of eq. (6) becomes $x_2(t) \approx \int G_1(t)dt$. This implies that the dynamics x_2 is nearly equivalent to the dynamics x_1. These considerations lead to the higher-order interactions between the dynamics x_1 and x_2. The formal expression (eq. (6)) of solution for x_2 is a recursive form so that one step of the iteration gives following representation:

$$x_2(t) = \int f_2 \left(\int f_2(x_2(t))\, dt + \int G_1(t)\, dt + c \right) dt + \int G_1(t)\, dt + c. \tag{7}$$

The eq. (7) implies that some nonlinearity of function f_2 can amplify the term $\int G_1(t)dt$ even if the term $\int G_1(t)dt$ is almost 0. Thus, the dynamics x_2 is controlled whether or not it depends upon the nonlinearity of function f_2. Hence we can formally write the solution of eq.

(7) as $x_2(t) = M_2(x_1(t))$, from which easily seen that the dynamics x_1 controls the dynamics of x_2.

Based on eq. (1) we can see that heterogeneous dynamics can appear from the separation to subsystems. Let $x(t)$ be $x(t) = x_1(t) + x_2(t)$, then eq. (1) becomes

$$\frac{d}{dt}(x_1 + x_2) = f(x_1 + x_2) = f_1(x_1) + f_2(x_2) + f_{12}(x_1, x_2), \tag{8}$$

where we divide $f(x_1 + x_2)$ to three terms as shown in the previous example. We furthermore separate eq. (8) two sets of differential equations as below,

$$\frac{d}{dt}x_1 = f_1(x_1) + \eta_1(x_1, x_2; \mu_{(1,2)}), \tag{9}$$

$$\frac{d}{dt}x_2 = f_2(x_2) + \eta_2(x_1, x_2; \mu_{(1,2)}), \tag{10}$$

where $\eta_1(x_1, x_2; \mu_{(1,2)})$ and $\eta_2(x_1, x_2; \mu_{(1,2)})$ satisfies the relation

$$\eta_1(x_1, x_2; \mu_{(1,2)}) + \eta_2(x_1, x_2; \mu_{(1,2)}) = f_{12}(x_1, x_2), \tag{11}$$

and those functions produce the same time development as that originally given by entire system equation denoted by eq. (1). The functions $\eta_1(x_1, x_2; \mu_{(1,2)})$ and $\eta_2(x_1, x_2; \mu_{(1,2)})$ are changed when the mode of realized the dynamics in original (entire) system changes. Thus, we introduce the parameter $\mu_{(1,2)}$ to represent the mode of dynamics. The mode of dynamics is related to the non-interacting version of the system, namely, the solutions of the differential equationsand $(d/dt)x_1 = f_1(x_1)$ and $(d/dt)x_2 = f_2(x_2)$. When these equations are nonlinear ones, many different modes of dynamics are expected. Therefore the interacting system described by the equations (9) and (10) is expected to show modulated modes of changing dynamics during the temporal development of the variables.

Since some modes yield negligibly small $\eta_1(x_1, x_2; \mu_{(1,2)})$ or $\eta_2(x_1, x_2; \mu_{(1,2)})$, the system described by equations (9) and (10) can realize control dynamics such as

$$x_2(t) = M_2(x_1(t); \mu_{(1,2)}) \text{ or } x_1(t) = M_1(x_2(t); \mu_{(1,2)}).$$

The alternative controlling is also expected in the dynamics. A memory effect can be realized by alternative controlling of dynamics. Causality between the control modes $M_2(x_1(t); \mu_{(1,2)})$ and $M_1(x_2(t); \mu_{(1,2)})$ is possible. That is the mode μ^* gives negligible small interaction term $\eta_1(x_1, x_2; \mu^*)$ or $\eta_2(x_1, x_2; \mu^*)$, so that the corresponding control mode $x_2(t) = M_2(x_1(t); \mu^*)$ or $x_1(t) = M_1(x_2(t); \mu^*)$ appears after the dynamics mode μ^* appears.

The above discussions using mathematical equations are directly expanded to the case of dividing to many element dynamics by putting $x = \Sigma_j x_j$. A difficult issue in the case of division into many elements is that the interaction functions between elements become quite complicated. But the same matters considered for the two element division case can occur in the many elements case. So the issues discussed in the present section are general.

3. Control dynamics demonstration by a one dimensional mapping system

We have conceptually considered what control dynamics are. Now we show that the matters considered above can be revealed in simulation studies using a one dimensional mapping. The demonstrations of discrete dynamics using a one dimensional recursive map have meaning because any differential equations are solved discretely by computers except for simply solvable equations.

3.1 One dimensional map introduced

In the present paper we use the one parameter interval map f:I→I shown in Figure 1. The interval of the one dimensional (1–D) recursion map is $[-1, 1]$. Therefore $I \in [-1, 1]$. The interval map f introduced is a cubic function having oscillatory features. So we call this map a "oscillator map". The oscillator map has three fixed points, namely, $\{(-1, -1), (0, 0), (1, 1)\}$ or $\{(-1, 1), (0, 0), (1, -1)\}$, and has a parameter A. Since the oscillator map has square bounds $\{(-1, -1), (1, -1), (1, 1), (-1, 1)\}$, the permitted parameter range becomes $0 \leq A \leq 4$ which is similar to logistic map [8]. The actual forms of mapping are

$$f(x) = Ax^3 + (1-A)x = Ax(x^2-1) + x, \tag{12}$$

or

$$f(x) = -Ax^3 - (1-A)x = -Ax(x^2-1) - x. \tag{13}$$

We restrict the parameter range where $f(x)=0$ has three different real solutions so that the parameter A is in the range $1 \leq A \leq 4$. There are positive and negative signs in the mappings. We therefore call these differences the p-type and the n-type.

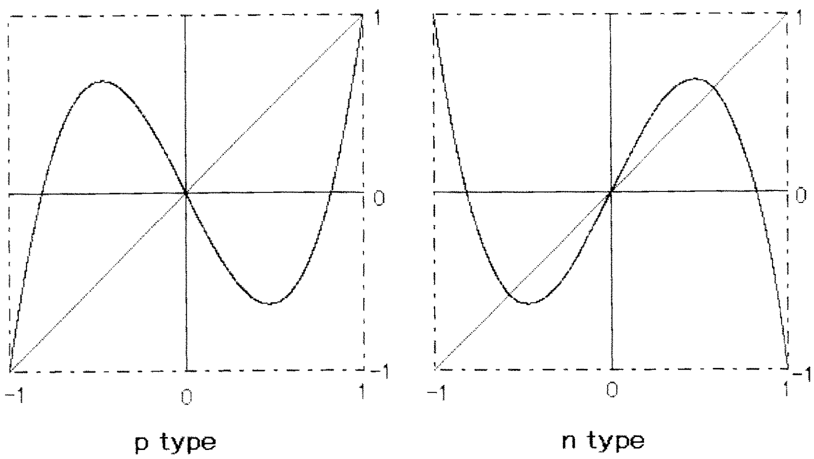

Fig. 1 Two type of oscillation maps.

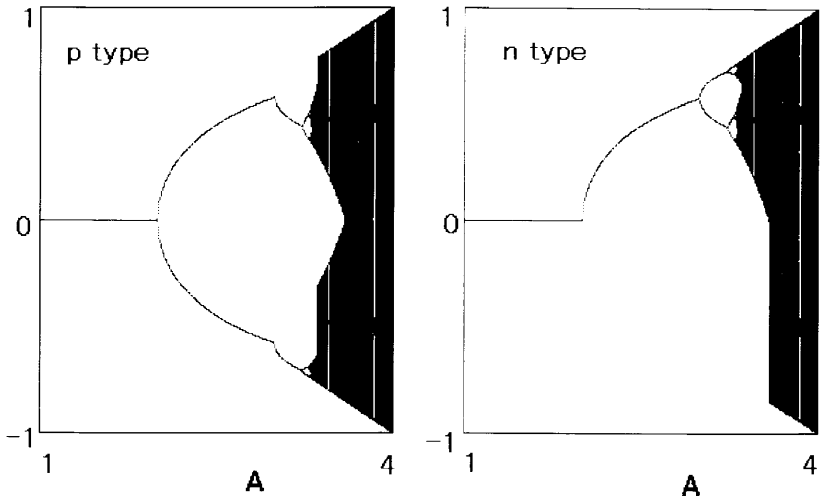

Fig. 2 Bifurcation Diagram for oscillation maps for an initial value of 0.4.

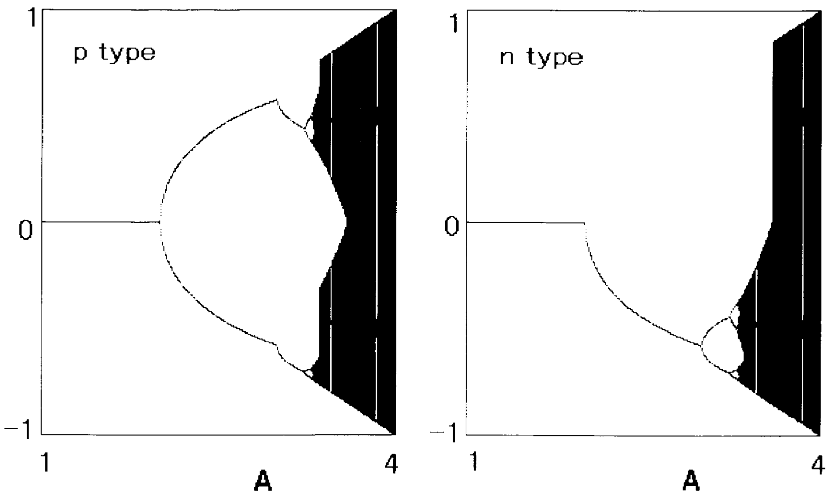

Fig. 3 Bifurcation Diagram for oscillation maps for an initial value of −0.3.

We next consider dynamics using oscillator maps $f(x)$. The temporal development variable x is governed by the following recurrence equation,

$$x(t+1)=f(x(t))=\begin{cases} Ax(t)(x^2(t)1)+x(t) \\ -Ax(t)(x^2(t)-1)-x(t) \end{cases}. \tag{14}$$

The bifurcation diagrams for parameter A are shown figures 2 to 4 following chaos studies [8]. Figures 2–4 are depicted using different initial values. As seen from figures 2–4, the dynamics of the oscillator map show partly different bifurcation features depending on the ini-

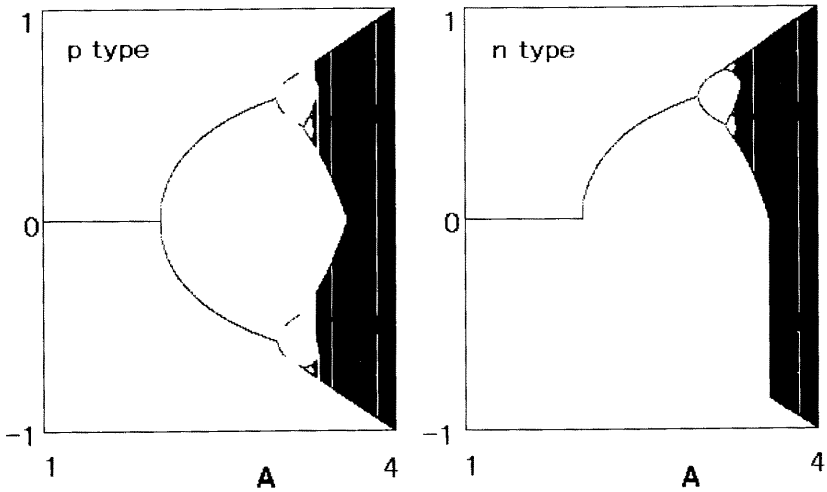

Fig. 4 Bifurcation Diagram for oscillation maps for an initial value of 0.00003.

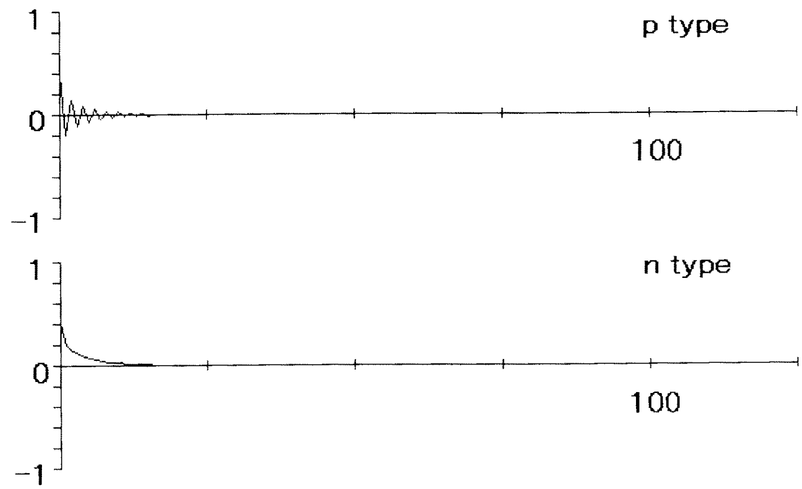

Fig. 5 Rapid damping example of the oscillation map.
The parameter and the initial value are A = 1.8 and $x(0) = 0.4$.

tial value. The dynamics of the oscillator map therefore has a basin structure. The basins [9] appear in the parameter range of about 3 to 3.3 for the p-type and about 2.3 to 3.6 for the n-type.

The typical modes for dynamics by p- and n-types of oscillator maps are shown figures 5 to 12. Figures 5 and 6 show the damped oscillations. Figure 5 shows rapid damping while figure 6 shows long-term damping. A period 4 oscillation is shown in figure 7. In figure 8, p-type dynamics show the rapid oscillation modulating a slow wavy oscillation, while n-type

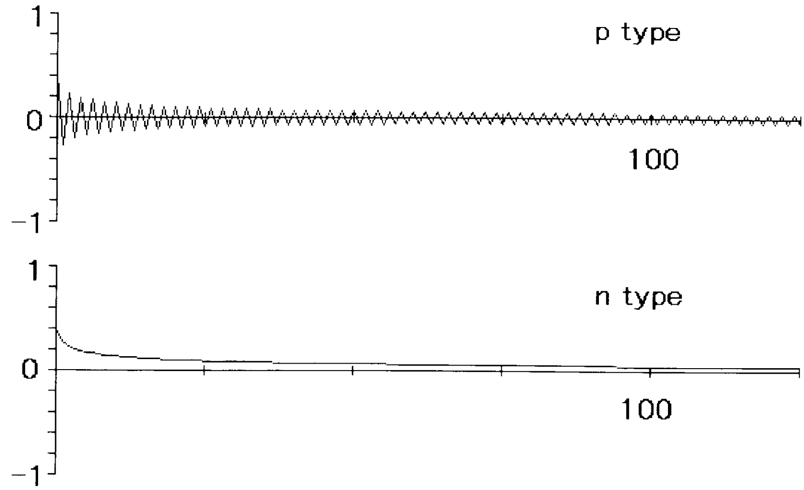

Fig. 6 Slow damping case of the oscillation map.
The parameter and the initial value are A = 2.0 and $x(0) = 0.4$.

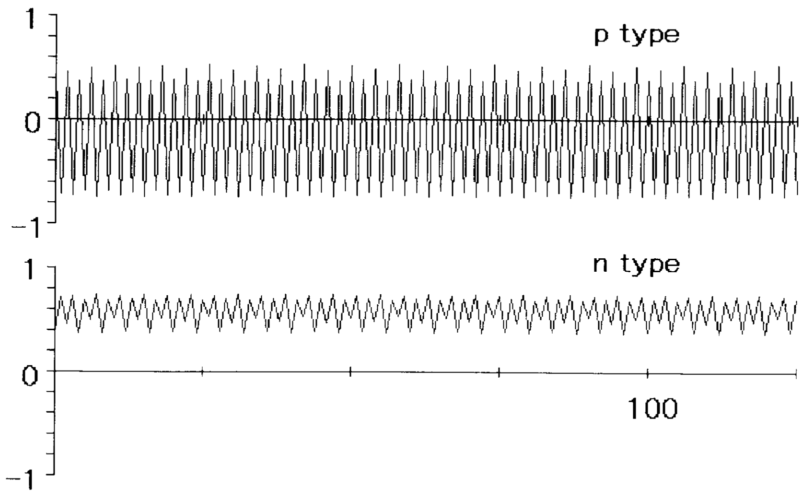

Fig. 7 Example of period-4 oscillation.
The parameter and the initial value are A = 3.3 and $x(0) = 0.4$.

dynamics show somewhat chaotic alternation of two different types of oscillations. Figures 9, 11, and 12 show the chaotic behaviors. The flip-flop changing of oscillations in positive and negative value regions is seen on n-type dynamics shown in figure 9. Figure 10 shows longer period oscillation. The n-type oscillation appeared in figure 10 asymptotes to a regular oscillation spanning entire range. There are different oscillation modes so that we hypothesize that these differences are states of the system. These states are brought by parameter changes of the mapping.

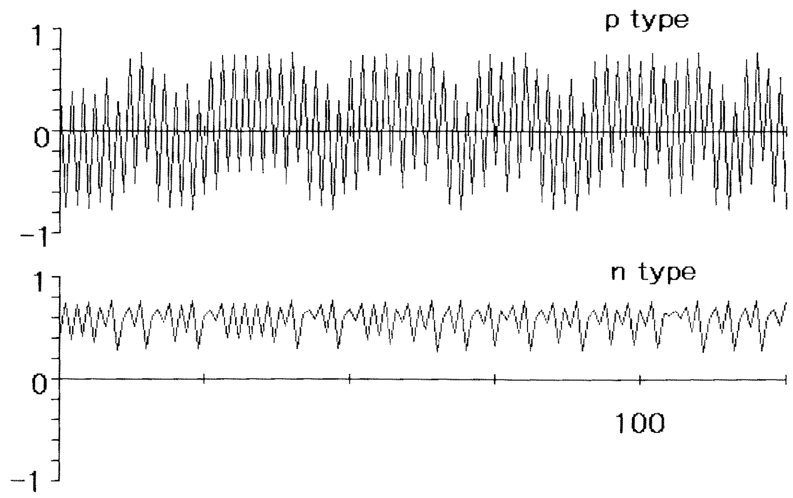

Fig. 8 Modulated oscillations with weak chaotic behavior.
The parameter and the initial value are A = 3.4 and $x(0) = 0.4$.

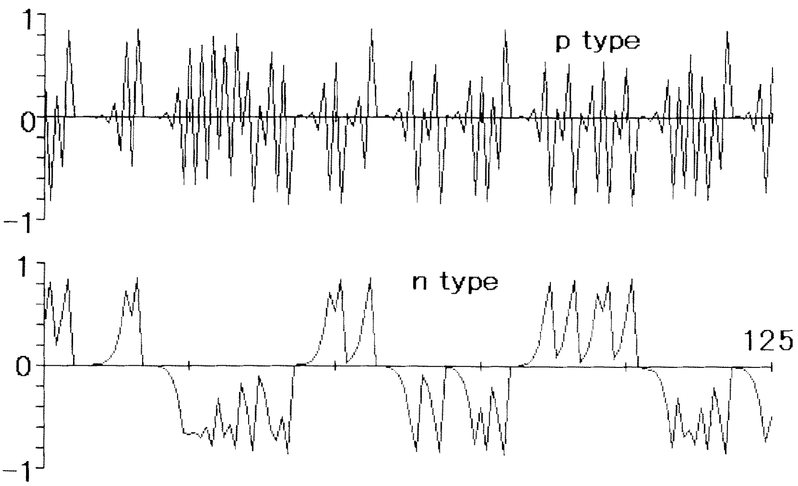

Fig. 9 Weak chaotic oscillations.
The parameter and the initial value are A = 3.6 and $x(0) = 0.4$.

3.2 Interaction systems of coupled three oscillators

Now we demonstrate how to realize control in the dynamics using coupled oscillator maps. The temporal developments of variables (x_1, x_2, x_3) are governed by the following set of recurrence equations:

$$x_1(t+1) = f(x_1(t); A_1) + \varepsilon_1 \eta(x_1(t), x_2(t), x_3(t)), \tag{15}$$

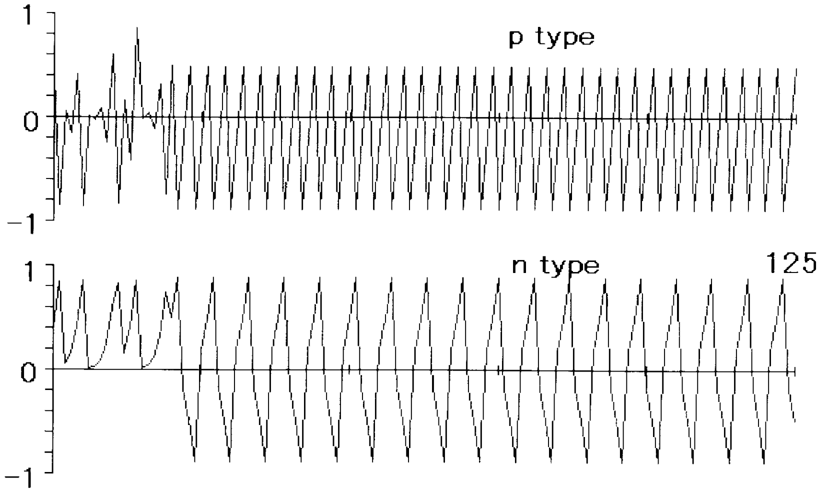

Fig. 10 Oscillation appeared in the bifurcation parameter of a window.
The parameter and the initial value are A = 3.7 and $x(0) = 0.4$.

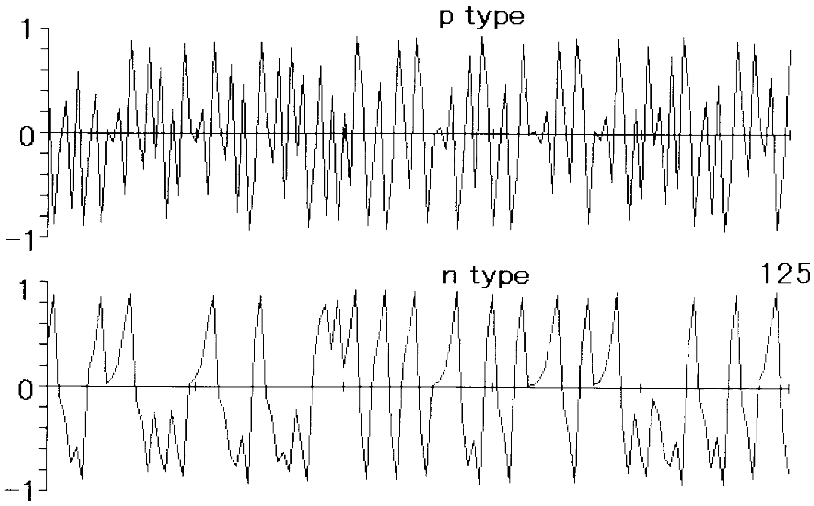

Fig. 11 Stronger chaotic oscillations.
The parameter and the initial value are A = 3.8 and $x(0) = 0.4$.

$$x_2(t+1) = f(x_2(t); A_2) + \varepsilon_2 \eta(x_1(t), x_2(t), x_3(t)), \tag{16}$$
$$x_3(t+1) = f(x_3(t); A_3) + \varepsilon_3 \eta(x_1(t), x_2(t), x_3(t)), \tag{17}$$

where $\eta(x_1, x_2, x_3)$ denotes interaction among three oscillators, and ε_1, ε_2, and ε_3 are switching factors such that $\varepsilon_k = 1$ ($k = 1, 2, 3$) for interaction on and $\varepsilon_k = 0$ ($k = 1, 2, 3$) for interaction off. Here we take two type interactions into account, namely, the average of the three varia-

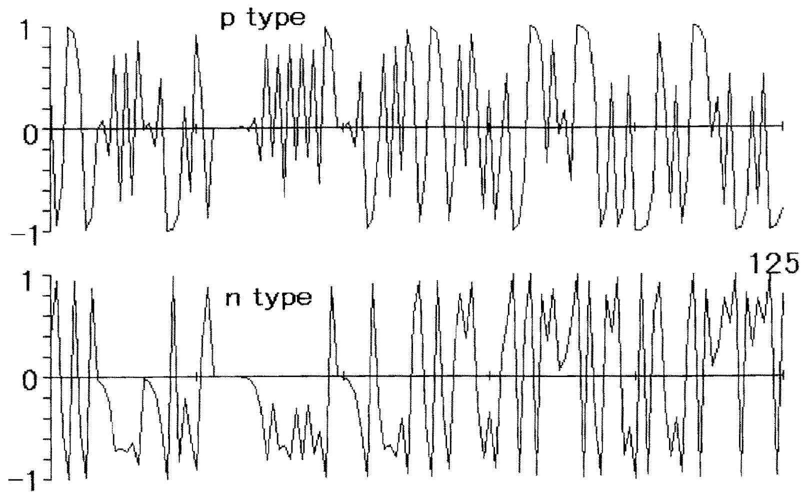

Fig. 12 Fully-developed chaotic oscillations.
The parameter and the initial value are A = 3.99 and $x(0) = 0.4$.

bles like the coupled map lattice [10], and the triple product of three variables. Those are expressed as follows:

$$\eta(x_1(t), x_2(t), x_3(t)) = \frac{1}{3}(x_1(t) + x_2(t) + x_3(t))$$

or

$$\eta(x_1(t), x_2(t), x_3(t)) = x_1(t)x_2(t)x_3(t). \qquad (18)$$

Using these two types of interactions, numerical studies are carried out on possible interaction schemes for a system of three 1–D oscillators, i.e., (1) two independent oscillators effect upon one servant oscillator, (2) effect of one independent oscillator upon two coupled oscillators, and (3) three coupled oscillators. For each of those three schemes, two interaction types are applied. The results are shown below.

Oscillator 3 behaviors affected by two input oscillators 1 and 2

The case considered here is obtained by putting $\varepsilon_1 = 0$, $\varepsilon_2 = 0$, $\varepsilon_3 = 1$ in equations (15)–(17). We can see how the states of input two oscillators compete. The interaction couples two inputs, and affects the state of third oscillator so that the state of oscillator 3 has an important roll organizing the mode.

I. Triple product interaction

The results are shown in figures 13 to 16. The dependency of affected oscillator states

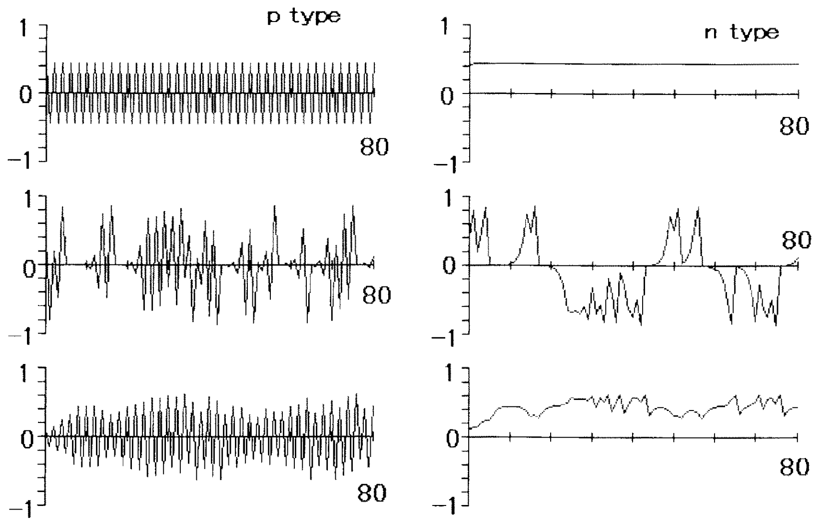

Fig. 13 Two input competition between steady state and weak chaos, for the triple product interaction. A1 = 2.5, A2 = 3.6, A3 = 2.5, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.1$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

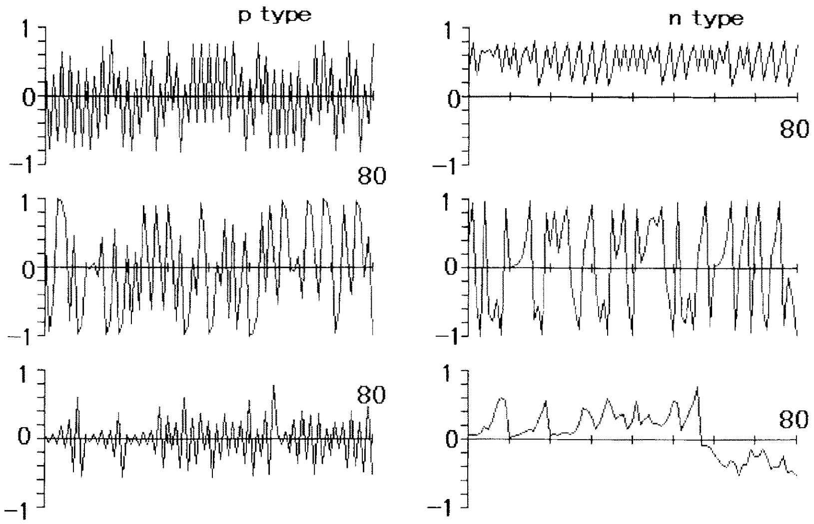

Fig. 14 Two input competition between weak chaos and fully-developed chaos, for the triple product interaction. A1 = 3.5, A2 = 3.99, A3 = 2.5, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.005$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

can be seen from figures 13 to 15. In those figures, the affected oscillator 3 is fixed at the parameter value A = 2.5. The parameter A = 2.5 generates period-two oscillation for the p-type and also when fixed at a value of about 0.45 for the n-type. Figure 16 shows the case where oscillator 3 is in the state of period-4 oscillations for both types of oscillators. The

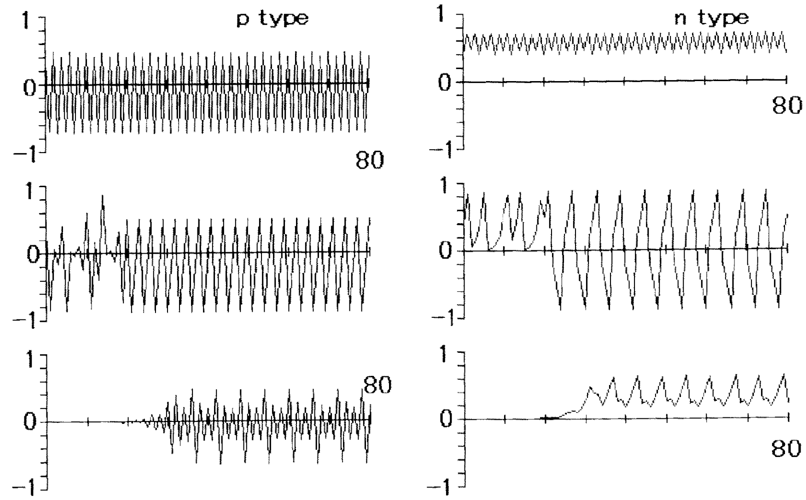

Fig. 15 Two input competition between period 4 oscillations and oscillations in window. Triple product interaction. A1 = 3.25, A2 = 3.7, A3 = 2.5, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.0001$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

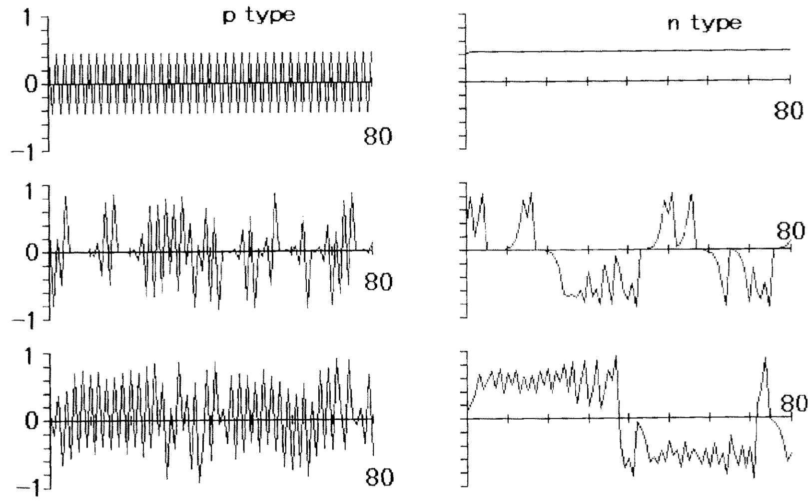

Fig. 16 Another state of the affected oscillator for the figure 13 case. Triple product interaction. A1 = 2.5, A2 = 3.6, A3 = 3.3, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.1$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

parameters of input oscillators 1 and 2 are the same values as those in figure 13. Comparing the two cases, the period-4 case of the affected oscillator 3 is more chaotic than the period-2 case. Since the interaction is the triple product $x_1x_2x_3$, the behavior of oscillator 3 is not a simple product of the input oscillator variables, but similar to the product of those two varia-

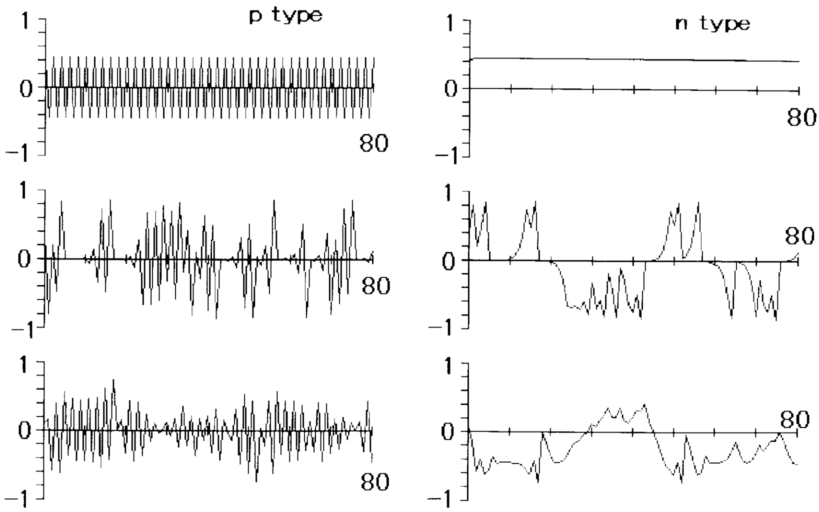

Fig. 17 Two input competition between steady state and weak chaos. Averaged interaction.
A1 = 2.5, A2 = 3.6, A3 = 2.5, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.1$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

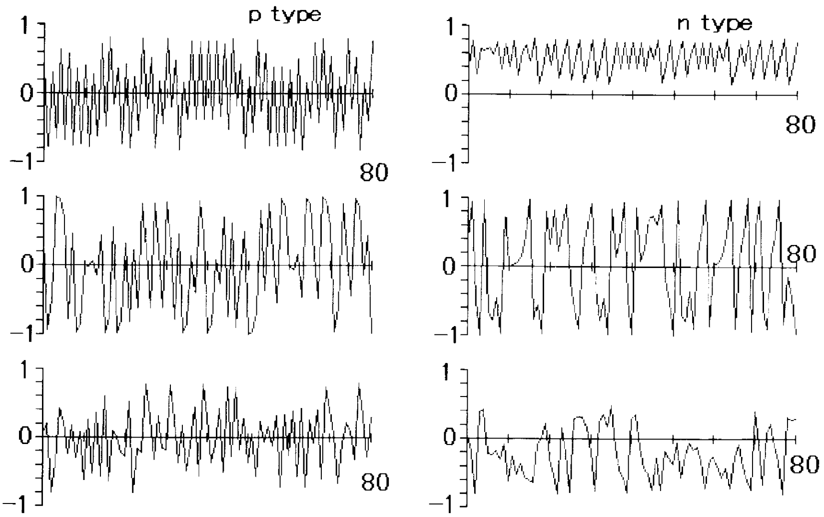

Fig. 18 Two input competition between weak chaos and fully-developed chaos. Averaged interaction.
A1 = 3.5, A2 = 3.99, A3 = 2.5, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.005$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

bles.

II. Averaged interaction

Figures 17–19 are the results for parameters corresponding to those of triple product interactions illustrated above. Thus, we can see how the affected oscillator state depends upon

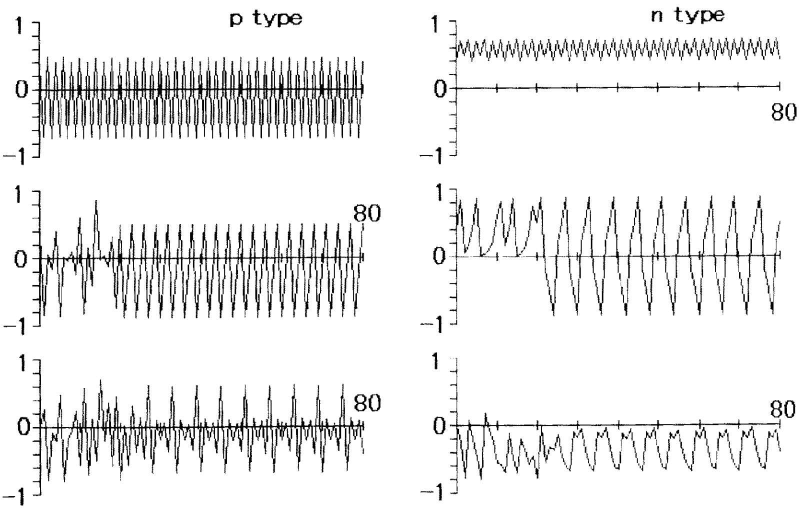

Fig. 19 Two input competition between period 4 oscillations and oscillations in window. Averaged interaction.
A1 = 3.25, A2 = 3.7, A3 = 2.5, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.0001$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

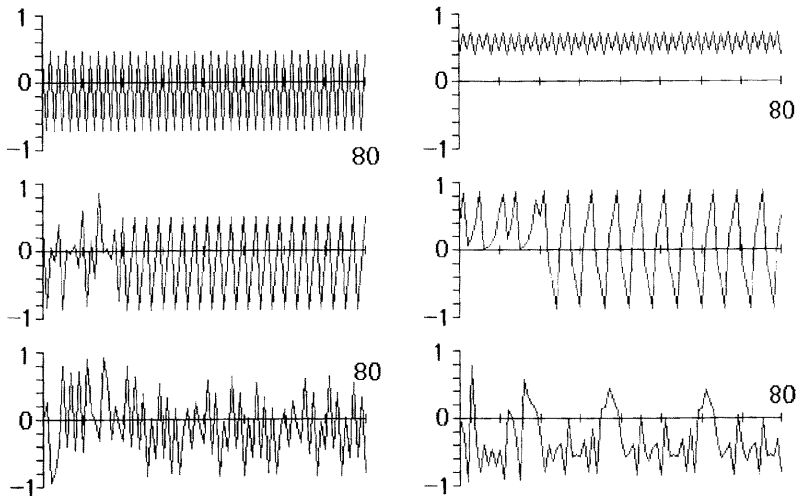

Fig. 20 Another state in effected oscillator corresponding to figure 19 case. Averaged interaction.
A1 = 3.25, A2 = 3.7, A3 = 3.2, and $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.0001$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

the states of the affecting oscillators. The triple oscillator system is unstable for the parameters (A_1, A_2, A_3) corresponding to those for figure 16. So we present the result for another parameter set in figure 20.

Comparing the numerical results between cases for triple product interaction and those for averaged interaction, we see that the additive interaction gives a more direct effect to oscillator 3 than the product type interaction. The triple product interaction is thus rather stable compared to the averaged interaction. As illustrated by the figures, the averaged interaction just generates behavior resembling amplitude modulation.

The oscillator 1 controls the coupled system of oscillators 2 and 3

This model is given by equations with the switching parameter values $\varepsilon_1 = 0$, $\varepsilon_2 = 1$, $\varepsilon_3 = 1$ for interactions. The states of the coupled oscillators are period two oscillation (oscillator 2) and weak chaotic oscillation (oscillator 3). The coupled oscillators are controlled by the input from oscillator 1. The coupling effects are different for the triple product interaction and averaged interaction.

I. Triple product interaction

The temporal developments of variables are shown in figures from 21 to 23. The behavior of coupled oscillators in the case of triple product interaction is well controlled by the input oscillator 1. If the controller (oscillator 1) is in the parameter region of period 2 oscillation, the controlled coupled oscillators are stabilized. When the controller is in the period-4 region, oscillator 2 of coupled system shows a small amplitude of period 2 oscillation in p-type mapping, moving rapidly down to a beating state in n-type mapping, and oscillator 3 shows somewhat disordered oscillations. As seen from figure 23, chaotic ordered regulation may be possible in triple product interaction when the controller dynamics becomes weakly chaotic.

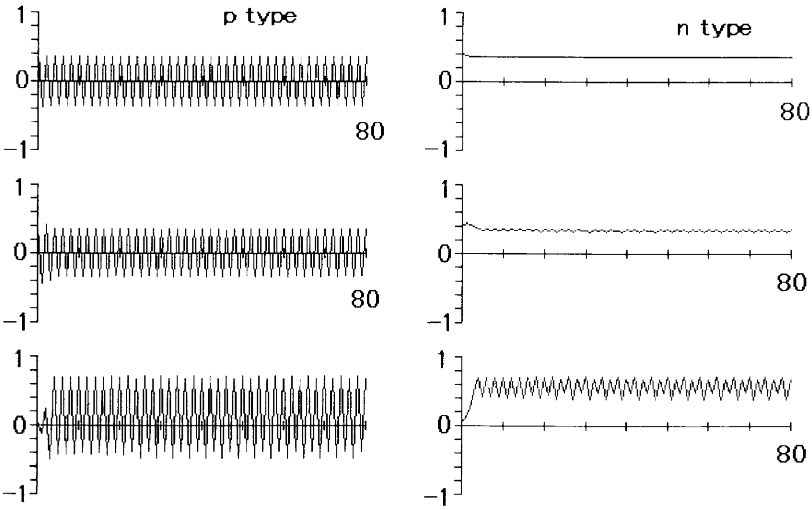

Fig. 21 Oscillator 1 controls coupled oscillators 2 and 3 for parameters A1 = 2.2, A2 = 2.5, A3 = 3.4. Triple product interaction, and initial conditions $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.05$. (upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

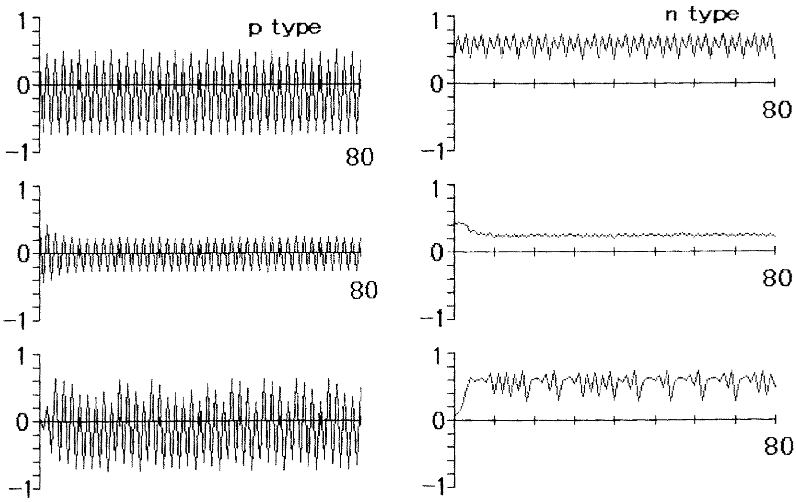

Fig. 22 Oscillator 1 controls coupled oscillators 2 and 3 for A1 = 3.3, A2 = 2.5, A3 = 3.4. Triple product interaction, and initial conditions $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.05$. (upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

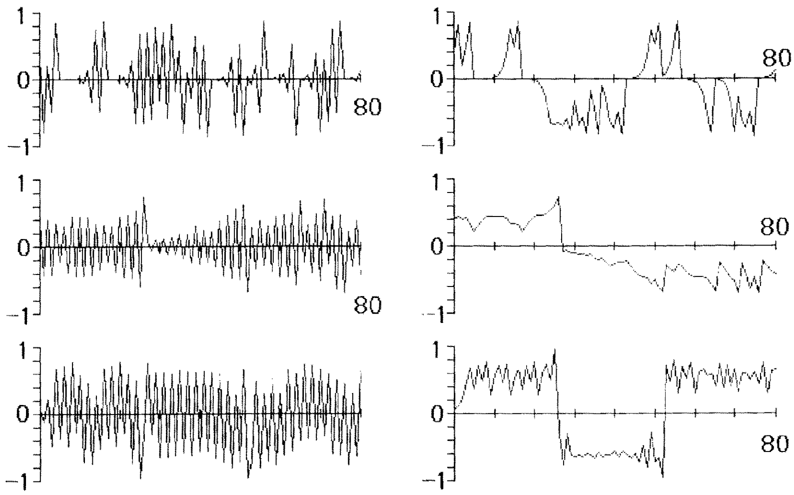

Fig. 23 Oscillator 1 controls coupled oscillators 2 and 3 for A1 = 3.6, A2 = 2.5, A3 = 3.4. Triple product interaction, and initial conditions $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.05$. (upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

II. Averaged interaction

Figures 24 to 26 show the time course of oscillations for the averaged interaction cases. The dynamics generated by averaged interactions more easily escape the interval [−1, 1] compared to those generated by the triple product interaction. Also, the dynamics generated

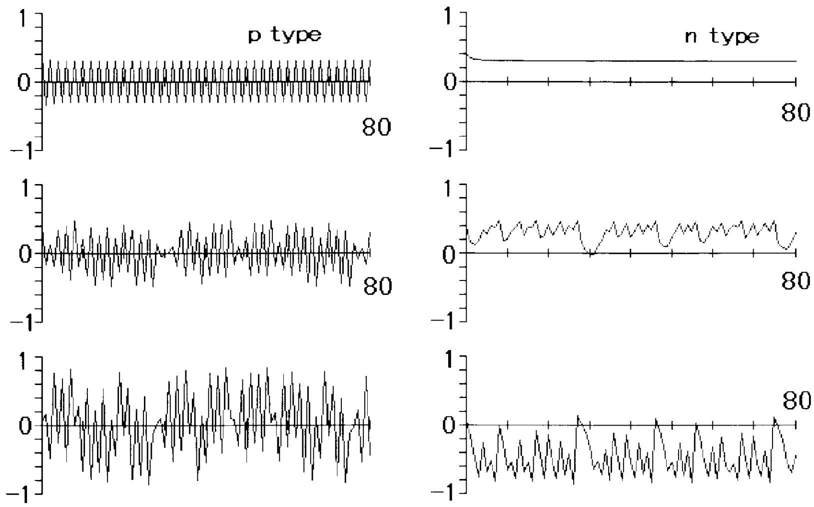

Fig. 24 Averaged interaction case for oscillator 1 controls coupled oscillators 2 and 3 with parameters A1 = 2.2, A2 = 2.5, A3 = 3.4. Initial conditions are $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.05$. (upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

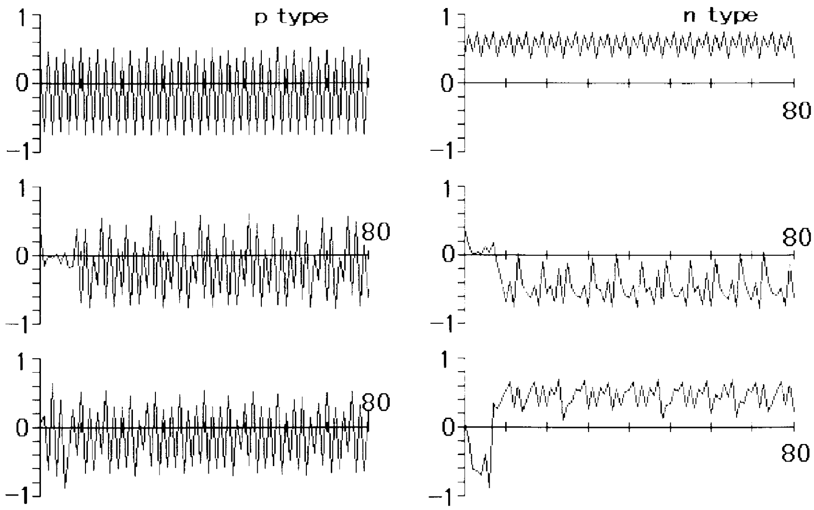

Fig. 25 Averaged interaction case for oscillator 1 controls coupled oscillators 2 and 3 with parameters A1 = 3.3, A2 = 2.5, A3 = 3.4. Initial conditions are $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.05$. (upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

by the averaged interaction have longer transients than those of the triple product interaction. The dynamics generated by the parameters shown in figure 26 will go out of the interval [− 1, 1] over 100 iteration steps in numerical calculation not shown in the present paper. In the case of figures 24 and 25, the three oscillators converge to their own steady state. Figure 27 shows the steady state convergence of the case shown in figure 24. This behavior is caused

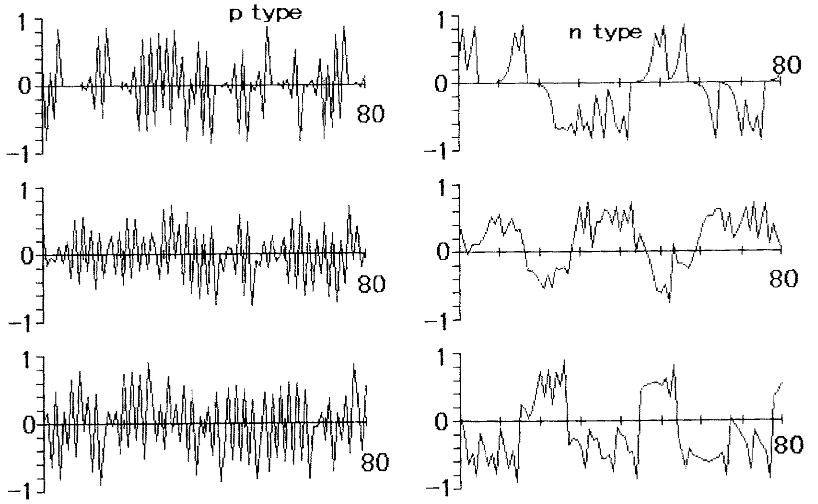

Fig. 26 Averaged interaction case for oscillator 1 controls coupled oscillators 2 and 3 with parameters A1 = 3.6, A2 = 2.5, A3 = 3.4. Initial conditions are $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.05$. (upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

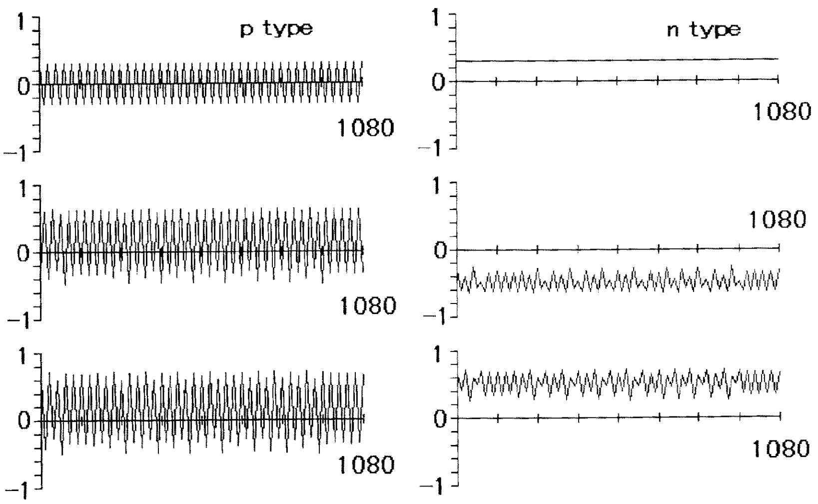

Fig. 27 Converged steady states in the case of figure 24. The iteration steps from 1000 to 1080 are depicted. (upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

by the controller feature, but the control effect requires many iteration steps to establish.

From figures 21–27 for cases where an oscillator controls a coupled oscillators system, the averaged interaction gives more chaotic dynamics than the triple product interaction. The system constructed with the triple product interaction is better for control since it rapidly approaches to steady states that can be realized. However, if the chaotic state of controller becomes strong, the controlled coupled oscillators go over to the divergence area. The averaged interaction is much more sensitive to the chaos. Overall, the triple product interaction reveals control dynamics.

All coupled triple oscillators

The temporal developments of all coupled oscillators are described by equations from (15) to (17) with the switching parameters $\varepsilon_1 = 1$, $\varepsilon_2 = 1$, $\varepsilon_3 = 1$. This system is not a control system in common sense, but the system may be regarded as an inner control system. We can discuss memory effects for the all the coupled oscillators.

I. Triple product interaction

Figures 28 and 29 present the case where all triple oscillators are coupled. The oscillator 3 in figure 29 is almost zero, but this oscillator goes to its own state of finite amplitude as shown in figure 30. This behavior may be called an induction phenomenon. The behavior is caused by a lengthy escape from an unstable fixed point. This is similar to tangential bifurcations. In this case the local area of the mapping is very close to the line given by $f(x) = x$ like the tangential bifurcation in intermittency [11,12]. The triple product interaction creates rather regulated behavior even though the system includes an oscillator that is weakly chaotic. Thus, we can understand that a system composed of oscillators having triple product interaction can realize a good memory effects.

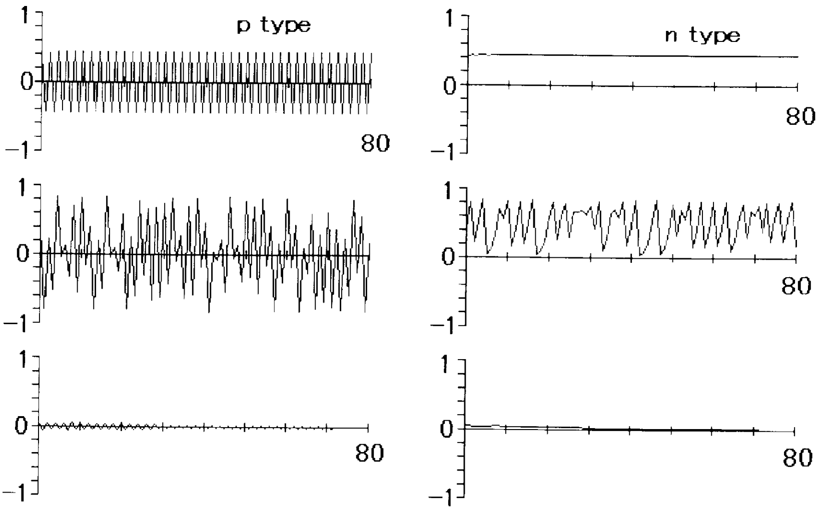

Fig. 28 States of triply coupled oscillators with triple product interaction for $A_1 = 2.5$, $A_2 = 3.6$, $A_3 = 2.2$. Initial conditions are $x_1(0) = 0.4$, $x_2(0) = 0.4$, $x_3(0) = 0.05$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

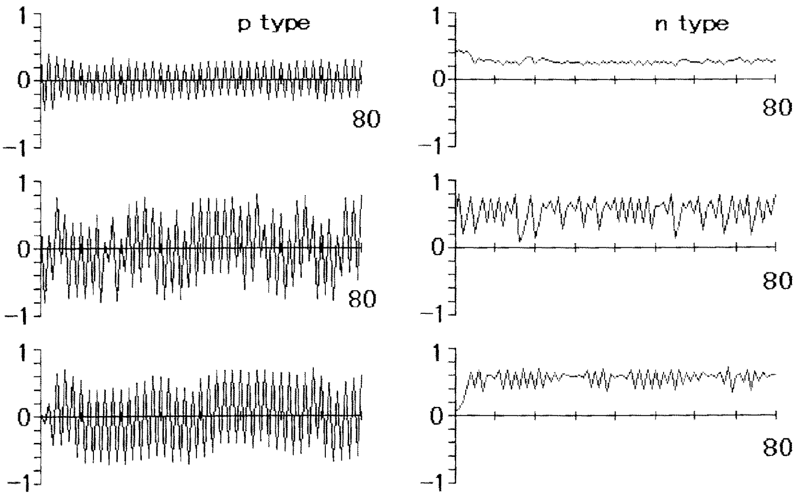

Fig. 29 States of triply coupled oscillators with triple product interaction for $A_1=2.5$, $A_2=3.6$, $A_3=3.3$. Initial conditions are $x_1(0)=0.4$, $x_2(0)=0.4$, $x_3(0)=0.05$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

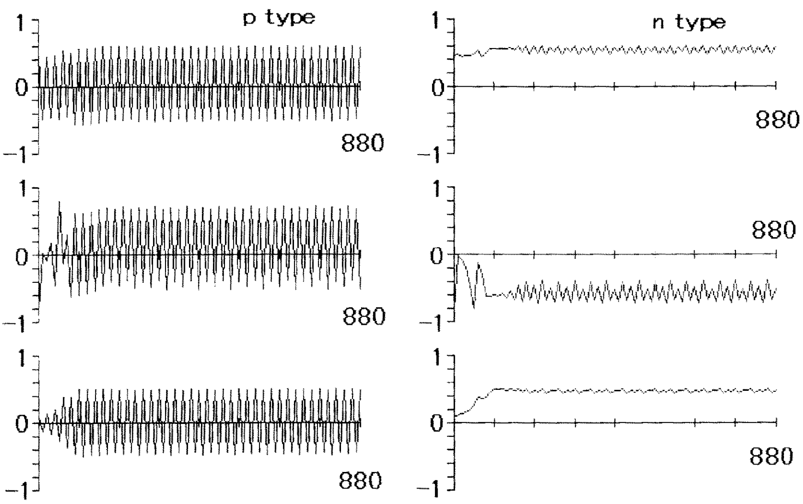

Fig. 30 Later iteration states for the case shown in figure 28.
The amplitude of oscillator 3 is grows over 800 step iterations to finite size, and each oscillator realizes its own steady state.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

II. Averaged interaction

The all oscillators coupled case for the averaged interaction shows different properties to those of the triple product interaction. The differences are easily seen by comparing figures 28 and 29 with figures 31 and 32. This system may be said to have chaotic memory, but is less

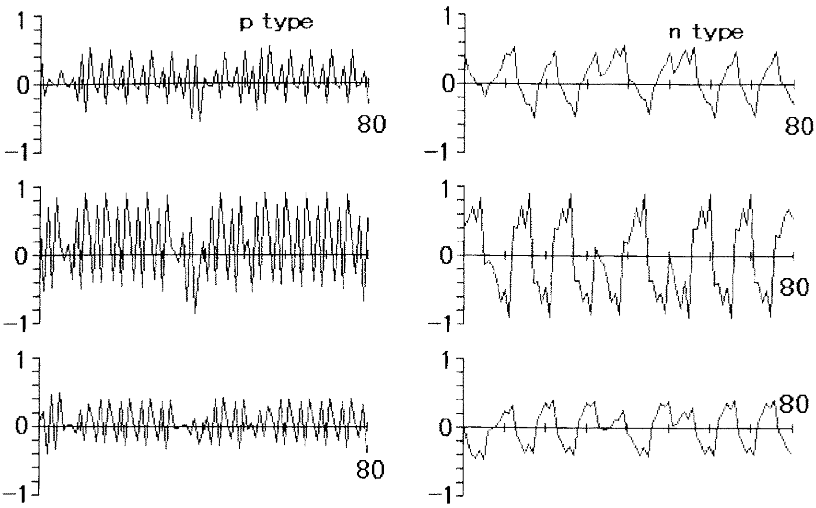

Fig. 31 States of triply coupled oscillators with the averaged interaction for $A_1=2.5$, $A_2=3.6$, $A_3=2.2$.
Initial conditions are $x_1(0)=0.4$, $x_2(0)=0.4$, $x_3(0)=0.05$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

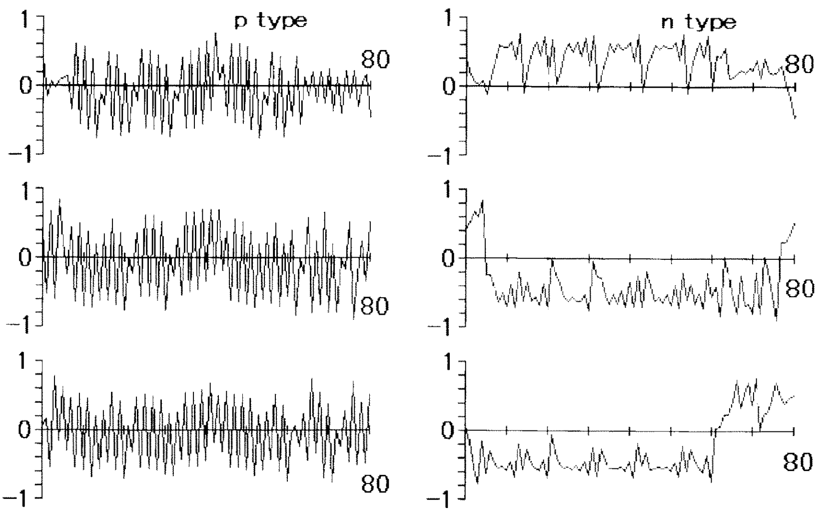

Fig. 32 States of triply coupled oscillators with the averaged interaction for $A_1=2.5$, $A_2=3.6$, $A_3=3.3$.
Initial conditions are $x_1(0)=0.4$, $x_2(0)=0.4$, $x_3(0)=0.05$.
(upper: oscillator 1, middle: oscillator 2, bottom: oscillator 3)

stable to control. This kind of system will be useful for searching memory systems.

4. Summary and discussions

We considered what control dynamics is. Usually control systems are studied by using

Laplace transformation for composed elements of a control system [13], namely, transfer functions. Our approach to control is from the viewpoint of dynamics in space-time. Then the structure of the control system is quite obscure or dissolved. The principal requirement for the control is causality for a sequenced of events. The concept is applicable for any system where some quantities describing the system develop temporally. Hence we regard memory processes as control dynamics established in a system.

We demonstrate the required concepts for control dynamics using an oscillation map. The oscillation map is the cubic function with three fixed points $(-1, -1)$, $(0, 0)$, and $(1, 1)$ for p-type, and $(-1, 1)$, $(0, 0)$, and $(1, -1)$ for n-type. This map is a parameter one-dimensional map, and has a basin for bifurcation in the parameter region shown in section 3. We introduced two types of interactions, the average of the input variables and the product of input variables. We discovered that averaged input easily causes chaotic behavior in the affected system, while the product input can reveal rather stable behaviors in the affected system. We showed that a controller in the system with triple product interaction realizes control dynamics.

We expect that the product type interaction will be useful for the processing of visual systems where the higher order correlation plays an important roll [14]. Of particular interest is that humans have been shown to use measures related to locally computed four-order correlations between pixels to discriminate textures [14,15]. Those studies employed so called *isotrigon* textures, ensemble averages of which have third-order correlation functions that are everywhere indistinguishable from 0. Since humans can easily learn to discriminate some of these classes of textures, by simply studying a few examples, it is clear that we must be using measures related to 4^{th} and higher order correlations to distinguish them. Many of these patterns are generated by cellular automata using the triple product of pixels in a recursion process [14,16]. It is expected that the 1-D map systems described here might be able to discriminate isotrigon textures, and so perhaps provide some insight into the neuro-dynamic process by which we establish memories of salient features of these textures for the purpose of discriminating classes of these textures, and thus emulating human visual performance.

In reference to visual cognition, the relationship between functions that cause dynamics will form what the visual cognitive process is. Probably the relationship among functions will be embedded into the dynamics, so that the heterogeneous structure of a system will realize a kind of hierarchical dynamics. This subject will be the topic of a forthcoming study.

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