論文　**Original Paper**

On the extensions of the cyclic p-groups

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Abstract: Let ϕ be a faithful irreducible character of the cyclic group C_n of order p^n, where p is an odd prime. We study the p-group G containing C_n such that the induced character ϕ^G is also irreducible. The purpose of this paper is to study the subgroup $N_G(N_G(N_G(C_n)))$ of G.

Keywords: p-group, extension, irreducible induced character, faithful irreducible character

1. Introduction

Let G be a finite group. We denote by $\mathrm{Irr}(G)$ the set of complex irreducible characters of G and by $\mathrm{FIrr}(G)$ ($\subset \mathrm{Irr}(G)$) the set of faithful irreducible characters of G.

Let p be a prime. For a non-negative integer n, we denote by C_n the cyclic group of order p^n. Further, we denote by Q_n, D_n and SD_n the generalized quaternion group, the dihedral group of order 2^{n+1} ($n \geqq 2$) and the semidihedral group of order 2^{n+1} ($n \geqq 3$), respectively.

When G is a p-group, for any $\chi \in \mathrm{Irr}(G)$, there exists a subgroup H of G and a linear character ϕ of H such that $\phi^G = \chi$. If we set $N = \mathrm{Ker}\ \phi$, then $N \lhd H$ and ϕ is a faithful irreducible character of $H/N \cong C_n$, for some non-negative integer n. In this paper, we will consider the case when $N = 1$, that is, ϕ is a faithful linear character of $H \cong C_n$.

We consider the following:

Problem A. *Let p be an odd prime, and ϕ be a faithful irreducible character of C_n. Determine the p-group G such that $C_n \subset G$ and the induced character ϕ^G is also irreducible.*

Since all the faithful irreducible characters of C_n are algebraically conjugate to each other, the irreducibility of ϕ^G ($\phi \in \mathrm{FIrr}(C_n)$) is independent of the choice of ϕ, and depends only on n.
This problem has been solved in each of the following cases:

(1) $C_n \lhd G$ ([2]),

(2) G has a subgroup H containing C_n such that $C_n \lhd H$ and $[G: H] = p$ ([5]).

See also [8] and [9].

On the other hand, when $p = 2$, Yamada and Iida [3] considered the following:

Problem B. *Let ϕ be a faithful irreducible character of H, where $H = Q_n$ or D_n or SD_n. Determine the 2-group G such that $H \subset G$ and the induced character ϕ^G is also irreducible.*

Yamada and Iida [3] solved Problem B in the case when $[G; H] = 2$ or 4 and we have solved it when $[G; H] = 8$ ([6]) for all $H = Q_n$ or D_n or SD_n.

Moreover, we have recently solved Problem B completely ([7]). In [7], we showed that

$$G = N_G(H) \text{ or } N_G(N_G(H)),$$

for all $H = Q_n$ or D_n or SD_n, if G satisfies the conditions of Problem B. Here, as usual, $N_G(H)$ and $N_G(N_G(H))$ are the normalizers of H and $N_G(H)$ in G, respectively. This means that, if we define subgroups of G by

$$M_1 = N_G(H), \text{ and } M_{i+1} = N_G(M_i), \text{ for } i \geqq 1,$$

then

$$H \subset M_1 \subset M_2 = M_3 = M_4 = \cdots = G,$$

for all $H = Q_n$ or D_n or SD_n.

In this paper, we consider Problem A. We also define subgroups of G by

$$N_1 = N_G(C_n), \text{ and } N_{i+1} = N_G(N_i), \text{ for } i \geqq 1.$$

The purpose of this paper is to study the group $N_3 = N_G(N_G(N_G(C_n)))$.

Throughout this paper, $\mathbf{Z}$ and $\mathbf{N}$ denote the set of rational integers and the natural numbers, respectively.

2. Statements of the results

For the rest of this paper, we assume that p is an odd prime.

First, we introduce the following groups:

(i) $G(n, m) = \langle a, b_m \rangle$ with
$$a^{p^n} = b_m^{p^m} = 1,\ b_m a b_m^{-1} = a^{1+p^{n-m}}, \quad (m \leqq n-1).$$

(ii) $G(n, m, t) = \langle a, b_m, v_t \rangle (\rhd G(n, m) = \langle a, b_m \rangle)$ with
$$a^{p^n} = b_m^{p^m} = 1,\ b_m a b_m^{-1} = a^{1+p^{n-m}},$$
$$v_t a v_t^{-1} = a^{1+p^{n-m-t}} b_m^{p^{m-t}},$$
$$v_t^{p^t} = b_m,\ v_t b_m v_t^{-1} = b_m \quad (4m \leqq n,\ 1 \leqq t \leqq m).$$

(iii) $G(n, 1, 1, 1) = \langle a, b_1, v_1, x \rangle (\rhd G(n, 1, 1) = \langle a, b_1, v_1 \rangle)$ with
$$a^{p^n} = b_1^p = 1,\ b_1 a b_1^{-1} = a^{1+p^{n-1}},\ v_1 a v_1^{-1} = a^{1+p^{n-2}} b_1,$$
$$v_1^p = b_1,\ v_1 b_1 v_1^{-1} = b_1,\ x a x^{-1} = a^{1+p^{n-3}} v_1,\ x^p = v_1,$$
$$x v_1 x^{-1} = v_1,\ x b_1 x^{-1} = b_1 \quad (7 \leqq n).$$

We can see that $G(n, m, t)$ (resp. $G(n, 1, 1, 1)$) is an extension group of $G(n, m)$ (resp. $G(n, 1, 1)$) by using Proposition 1 below:

Proposition 1. *Let N be a finite group such that $G \rhd N$*

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and $G/N=\langle uN\rangle$ *is a cyclic group of order* m. *Then* $u^m=c\in N$. *If we put* $\sigma(x)=uxu^{-1}$, $x\in N$, *then* $\sigma\in Aut(N)$ *and* (*i*) $\sigma^m(x)=cxc^{-1}$, $(x\in N)$ (*ii*) $\sigma(c)=c$.

Conversely, if $\sigma\in Aut(N)$ *and* $c\in N$ *satisfy* (*i*) *and* (*ii*), *then there exists one and only one extension group* G *of* N *such that* $G/N=\langle uN\rangle$ *is a cyclic group of order* m *and* $\sigma(x)=vxv^{-1}$ $(x\in N)$ *and* $v^m=c$.

Proof. For instance, see [10, III, § 7].

We now state the known results concerning Problem A.

Theorem 0.1 (Iida [2]). *Let* G *be a p-group which contains* C_n *as a normal subgroup of index* p^m. *Let* $\phi\in \mathrm{FIrr}(C_n)$. *Suppose that* $\phi^G\in \mathrm{Irr}(G)$. *Then* $m\leqq n-1$, *and* $G\cong G(n, m)$.

Corollary 0.2. *Let* G *be a p-group which contains* C_n. *Let* $\phi\in \mathrm{FIrr}(C_n)$. *Suppose that* $\phi^G\in \mathrm{Irr}(G)$. *Then* $N_1\cong G(n, m)$.

Theorem 0.3 ([5]). *Let* G *be a p-group which contains* C_n, *and let* $\phi\in \mathrm{FIrr}(C_n)$. *Suppose that* $[G: C_n]=p^{m+1}$, $\phi^G\in \mathrm{Irr}(G)$, *and* $n-3\geq 2m$. *Further, suppose that there exists a subgroup* H *of* G *such that* $H\rhd C_n$ *and* $[G: H]=p$. *Then*

(1) $G\cong G(n, m+1)$ *if* C_n *is a normal subgroup of* G.

(2) $G\cong G(n, m, 1)$ *if* C_n *is not a normal subgroup of* G.

Theorem 0.4 ([8]). *Let* p *be an odd prime. Let* G *be a p-group which contains* $C_n=\langle a\rangle$. *We assume that* $[G: C_n]\geq p^3$. *Define the subgroups of* G *by*

$N_1=N_G(C_n)$, and $N_{i+1}=N_G(N_i)$, for $i=1, 2$.

Let $\phi\in \mathrm{FIrr}(C_n)$ *and* $7\leqq n$. *Suppose that* $\phi^G\in \mathrm{Irr}(G)$, *and* $[N_1: C_n]=p$. *Then*

(1) $N_2/N_1\cong C_1$ and $N_2\cong G(n, 1, 1)$,

(2) $N_3/N_2\cong C_1$ and $N_3\cong G(n, 1, 1, 1)$.

Recently, we have determined the subgroup $N_2=N_G(N_G(C_n))$.

Theorem 0.5 ([9]). *Let* p *be an odd prime. Let* G *be a p-group which contains* $C_n=\langle a\rangle$. *Suppose that* $\phi^G\in \mathrm{Irr}(G)$ *for any* $\phi\in \mathrm{FIrr}(C_n)$. *Suppose further that* $4m\leqq n$, *where* $[N_1: C_n]=p^m$. *Then*

(1) $N_2=N_G(N_G(C_n))\cong G(n, m, t)$, *if* $[G: N_1]=p^t\leqq p^{m-1}$.

(2) $N_2=N_G(N_G(C_n))\cong G(n, m, m)$, *if* $[G: N_1]\geqq p^m$.

REMARK 1. Conversely, it is easy to see that the groups $G(n, 1, 1)$ and $G(n, 1, 1, 1)$ satisfy the condition (EX, C), which is defined in section 3 of this paper. Hence these groups satisfy the conditions of Problem A.

Our main theorem is the following:

Theorem. *Let* p *be an odd prime. Let* G *be a p-group which contains* $C_n=\langle a\rangle$. *Suppose that* $\phi^G\in \mathrm{Irr}(G)$ *for any* $\phi\in \mathrm{FIrr}(C_n)$. *Suppose further that* $6m\leqq n$, *where* $[N_1: C_n]=p^m$. *Then*

(1) $N_3/N_2=N_G(N_2)/N_2\cong C_r$ for some $r\in \mathbf{N}$, $r\leqq m$.

(2) *Suppose that* $N_2=\langle a, b, v\rangle$, *then there exists* $g\in N_3$ *such that* $N_3=\langle a, b, v, g\rangle$, *and* g *satisfies the following relations:*

$$gag^{-1}=a^{1+kp^{n-2m-r}}v^{p^{m-r}},\ gvg^{-1}=v,$$
for some $k\in \mathbf{Z}$, $(k, p)=1$.

For the rest of this section, we state some results concerning the criterion of the irreducibilities of induced characters and others, which we need in section 3.

We denote by $\zeta=\zeta_{p^n}$ a primitive p^nth root of unity. It is known that, for $C_n=\langle a\rangle$, there are p^n irreducible characters ϕ_ν $(1\leqq \nu\leqq p^n)$ of C_n:

$$\phi_\nu(a^i)=\zeta^{\nu i},\quad (1\leqq i\leqq p^n).$$

The irreducible character ϕ_ν is faithful if and only if $(\nu, p)=1$.

We will need the following result of Shoda (cf [1, p. 329]):

Proposition 2. *Let* G *be a group and* H *be a subgroup of* G. *Let* ϕ *be a linear character of* H. *Then the induced character* ϕ^G *of* G *is irreducible if and only if, for each* $x\in G-H=\{g\in G\,|\,g\notin H\}$, *there exists* $h\in xHx^{-1}\cap H$ *such that* $\phi(h)\neq\phi(x^{-1}hx)$. (*Note that, when* ϕ *is faithful, the condition* $\phi(h)\neq\phi(x^{-1}hx)$ *holds if and only if* $h\neq x^{-1}hx$).

Using this result, we have the following:

Proposition 3. *Let* $\langle a\rangle=C_n\subset G$, *and* ϕ *be a faithful irreducible character of* C_n. *Then the following conditions are equivalent:*

(1) ϕ^G *is irreducible*,

(2) *For each* $x\in G-C_n$, *there exists* $y\in\langle a\rangle\cap x\langle a\rangle x^{-1}$ *such that* $x^{-1}yx\neq y$.

DEFINITION. When the condition (2) of Proposition 3 holds, we say that G satisfies (EX, C).

Let H be a group. For a normal subgroup N of H, and any g, $h\in H$, we write

$$g\equiv h \pmod N,$$

when $g^{-1}h\in N$. For an element $g\in H$, we denote by $|g|$ the order of g.

3. Proof of Theorem

Let $N_1=N_G(C_n)$, $N_2=N_G(N_1)$, and $N_3=N_G(N_2)$.

Then, by [2], we an take an element $b_m\in N_1-C_n=\{g\in N_1\,|\,g\notin C_n\}$ such that

$$N_1=\langle a, b_m\,|\,a^{p^n}=b_m^{p^m}=1,\ b_mab_m^{-1}=a^{1+p^{n-m}}\rangle\cong G(n, m).$$

Hereafter, we use the notation b instead of b_m. These elements a and b satisfy the following

Lemma 1 ([9]) *Suppose that* $n\geqq 2m$, *then the following equalities hold for any* i, j, $k\in\mathbf{Z}$ *and* $x\in\mathbf{N}$.

(i) $ab^{p^{m-t}}\equiv b^{p^{m-t}}a \pmod{\langle a^{p^{n-t}}\rangle}$. $(0\leqq t\leqq m)$.

(ii) $ba^{p^m}b^{-1}=a^{p^m}$.

(iii) $b^j a^i b^{-j} = a^{i(1+jp^{n-m})}$.
(iv) $(a^i b^j)^x = a^{ix+ijp^{n-m}(x(x-1)/2)} b^{xj}$
(v) $(a^i b^{jp^{m-t}})^{p^t} = a^{ip^t}$ $(0 \leqq t \leqq m)$.
(vi) $(a^i b^{jp^{m-t}})^{1+kp^t} = a^{i(1+kp^t)} b^{jp^{m-t}}$ $(0 \leqq t \leqq m)$.

On the other hand, by [9], we can take $d \in \mathbf{Z}$, and $v_m \in N_2 - N_1 = \{g \in N_2 \mid g \notin N_1\}$ such that $(d, p) = 1$ and $a_1 = a^d$, b and v_m generate N_2. Moreover, $N_2 \cong G(n, m, m)$. That is,

$$N_2 = \langle a_1, b, v_m \rangle \cong G(n, m, m),$$

with

$$a_1^{p^n} = b^{p^m} = 1, \quad b a_1 b^{-1} = a_1^{1+p^{n-m}},$$
$$v_m a v_m^{-1} = a_1^{1+p^{n-2m}} b, \quad v_m^{p^m} = b, \quad v_m b v_m^{-1} = b.$$

Hereafter, we use the notation a (resp. v) instead of a_1 (resp. v_m).

Concerning to these elements a, b, v we can prove the following

Lemma 2 ([9]) *Suppose that $n \geqq 4m$, then the following equalities hold for any i, $j \in \mathbf{Z}$ and $x \in \mathbf{N}$,*
(i) $v a^{p^{2m}} v^{-1} = a^{p^{2m}}$.
(ii) $v^j a v^j = a^{1+jp^{n-2m}} b^j$.
(iii) $(a^i v^j)^x \equiv a^{ix+ijp^{n-2m}(x(x-1)/2)} b^{ij(x(x-1)/2)} v^{xj} \pmod{\langle a^{p^{n-m}} \rangle}$.
(iv) $(a^i v^j)^{p^m} \equiv a^{ip^m} b^j \pmod{\langle a^{p^{n-m}} \rangle}$.
(v) $(a^i v^j)^{p^{2m}} \equiv a^{ip^{2m}}$

.

Let f: $N_3 \to N_3/N_2$ be a natural epimorphism of groups. For $g \in N_3$, we write $o(g)$ for the order of $f(g)$ in N_3/N_2.

Define $p^{r_0} = max\{o(g) \mid g \in N_3\}$, and take an element $g_0 \in N_3$ such that $o(g_0) = p^{r_0}$. Hereafter we fix the element g_0. Then we can show the following:

Claim I. (i) $r_0 \leqq m$.

(ii) *For any $g \in N_3$, there exist integers s_1, k_1, j_1, satisfying $(k_1, p) = (j_1, p) = 1$, such that the following equalities hold:*

$$(a^{s_1} g) a (a^{s_1} g)^{-1} = a^{1+k_1 p^{n-2m-r}} v^{p^{m-r} j_1},$$
$$(a^{s_1} g) v (a^{s_1} g)^{-1} = v,$$

where $o(g) = p^r$.

Proof of Claim I. (i). Take an element $g \in N_3$. and let $o(g) = p^r$. We will show that $r \leqq m$.

By the same way as in the proof of [9], we can find $s_1 \in \mathbf{Z}$ such that $(a^{s_1} g) v (a^{s_1} g)^{-1} = v$. Set $g_1 = a^{s_1} g$, and write $g_1 a g_1^{-1} = a^i v^j$.

Since $g_1 a^{p^{2m}} g_1^{-1} = (a^i v^j)^{p^{2m}} = a^{ip^{2m}}$, by Lemma 2 (v), we must have $(p, i) = 1$.

Define the abelian subgroup M of G by

$$M = \langle a^{p^{n-2m}} \rangle \times \langle b \rangle.$$

It is easy to see that N_2/M is the abelian group. Further, since $6m \leqq n$, by our assumption, we have $g_1 a^{p^{n-2m}} g_1^{-1} = a^{ip^{n-2m}}$. So, we can see that $g_1 M g_1^{-1} = M$.

Since N_2/M is the abelian group, we have $va \equiv av$ (mod M). Using this relation repeatedly, we have

$$g_1^l a g_1^{-l} \equiv a^{i^l} v^{j(i^{l-1} + \cdot + i + 1)} \pmod{M},$$

for any $l \in \mathbf{N}$.

In particular,

$$g_1^{p^r} a g_1^{-p^r} \equiv a^{i^{p^r}} v^{j(i^{p^r-1} + \cdot + i + 1)} \pmod{M}.$$

But $g_1^{p^r} \in N_2$ and N_2/M is the abelian group, so,

$$g_1^{p^r} a g_1^{-p^r} \equiv a \pmod{M}.$$

Therefore we must have

$$i^{p^r} \equiv 1 \pmod{p^{n-2m}}, \qquad (1)$$

and

$$j(i^{p^r-1} + \cdot + i + 1) = j\left(\frac{i^{p^r} - 1}{i - 1}\right) \equiv 0 \pmod{p^m}. \qquad (2)$$

By (1), we can write as $i = 1 + i_1 p^t$, for some $i_1 \in \mathbf{Z}$ and $t \in \mathbf{N}$.

So,

$$j(i^{p^r-1} + \cdot + i + 1) = j\left(\frac{i^{p^r} - 1}{i - 1}\right) \equiv jp^r w, \qquad (3)$$

for some integer w, $(p, w) = 1$.

Suppose that $r \geqq m + 1$, then

$$j(i^{p^r-1} + \cdot + i + 1) \equiv 0 \pmod{p^m}.$$

This means that $g_1^{p^{r-1}} a g_1^{-p^{r-1}} \in M$ and $g_1^{p^{r-1}} \in N_2$, which contradicts our hypothesis that $o(g_1) = p^r$. Thus the proof of Claim I (i) is completed.

(ii). By (1) and (3), we can write as $i = 1 + k_1 \mathrm{p}^{n-2m-r}$, and $j = p^{m-r} j_1$ for some integers k_1 and j_1. We have

$$g_1 a g_1^{-1} = a^{1+k_1 p^{n-2m-r}} v^{p^{m-r} j_1}, \qquad (4)$$

and

$$g_1 v g_1^{-1} = v. \qquad (5)$$

Suppose that $(p, j_1) = p$, then by the equality

$$j(i^{p^{r-1}-1} + \cdot + i + 1) = j\left(\frac{i^{p^{r-1}} - 1}{i - 1}\right) = jp^{r-1} w_1, \qquad (6)$$

we can see that $g_1^{p^{r-1}} \in N_2$, which contradicts the hypothesis that $o(g_1) = p^r$. So, we must have $(p, j_1) = 1$.

Suppose that $(p, k_1) = p$. Write as $k_1 = k_2 p$, for some $k_2 \in \mathbf{Z}$. Since $n \geqq 6m$, we have

$$g_1 a^{p^{n-2m-r}} g_1^{-1} = a^{p^{n-2m-r}}.$$

Therefore

$$g_1^l \mathrm{a} g_1^{-1} = a^{1+lk_1 p^{n-2m-r}} v^{p^{m-r} j_1 l},$$

for any $l \in \mathbf{N}$.

In particular,

$$g_1^{p^{r-1}} a g_1^{-p^{r-1}} = a^{1+k_1 p^{n-2m-1}} v^{j_1 p^{m-1}} = a^{1+k_2 p^{n-2m}} v^{j_1 p^{m-1}}.$$

And

$$g_1^{p^{r-1}} a^{p^{m+1}} g_1^{-p^{r-1}} = a^{\mathrm{p}^{m+1}(1+k_1 p^{n-2m-1})} = a^{p^{m+1}(1+\mathrm{k}_2 p^{n-2m})}.$$

So, we can see that

$$g_1^{p^{r-1}} \langle a \rangle g_1^{-p^{r-1}} \cap \langle a \rangle = \langle a^{p^{m+1}} \rangle.$$

On the other hand, since

$$vav^{-1} = a^{1+p^{n-2m}} b,$$

we have

$$v a^{p^{m+1}} v^{-1} = a^{p^{m+1}(1+p^{n-2m})}.$$

So,

$$v^{-k_2}(g_1^{p^{r-1}} a^{p^{m+1}} g_1^{-p^{r-1}}) v^{k_2} = v^{-k_2}(a^{p^{m+1}(1+k_2 p^{n-2m})}) v^{k_2} = a^{p^{m+1}},$$

which contradicts our hypothesis that G satisfies (EX, C). Thus we must have $(k_1, p) = 1$. and we have completed the proof of (ii).

Claim II. *N_3 is generated by a, b, v and g_0.*

$$N_3 = \langle a, b, v, g_0 \rangle.$$

Proof of Claim II. Take an arbitrary element $h \in N_3$. Let $o(h) = p^{t_1}$. Then we can find $x_1 \in \mathbf{Z}$, such that $h_1 = a^{x_1} h$ satisfies the following equalities:

$$h_1 a h_1^{-1} = a^{1+d_1 p^{n-2m-t_1}} v^{d_2 p^{m-t_1}},$$

and

$$h_1 v h_1^{-1} = v,$$

where $(p, d_1) = (p, d_2) = 1$. Since $t_1 \leqq r_0$, we can take an element $u \in \langle a, b, v, g_0^{p^{r_0 - t_1}} \rangle$ such that

$$u^{-1}au = a^{1+e_1p^{n-2m-t_1}}v^{e_2p^{m-t_1}},$$

and

$$u^{-1}vu = v,$$

where $(p, e_1) = (p, e_2) = 1$. Let y be the integer satisfying $e_2y \equiv -d_2 \pmod{p^{m+t_1}}$, and set $u_1 = u^y$. Then

$$u_1^{-1}au_1 = a^{1+e_1yp^{n-2m-t_1}}v^{e_2yp^{m-t_1}} = a^{1+e_1yp^{n-2m-t_1}}v^{-d_2p^{m-t_1}},$$

and

$$u_1^{-1}vu_1 = v.$$

Therefore we have

$$u_1^{-1}(h_1ah_1^{-1})u_1 = u_1^{-1}(a^{1+d_1p^{n-2m-t_1}}v^{d_2p^{m-t_1}})u_1$$
$$= a^{1+(e_1y+d_1)p^{n-2m-t_1}} \in \langle \mathrm{a} \rangle.$$

This means that $u_1^{-1}h_1 \in N_G(C_n) = N_1$, so we have $h_1 \in \langle a, b, v, g_0 \rangle$. This complete the proof of Claim II.

Finally, we show the rest of the proof of the theorem. Let

$$g_0ag_0^{-1} = a^{1+k_0p^{n-2m-r_0}}v^{p^{m-r_0}j_0},$$

and

$$g_0vg_0^{-1} = v,$$

where $(p, k_0) = (p, j_0) = 1$. Since $(j_0, p) = 1$, we can find the integer c such that

$$j_0c \equiv 1 \pmod{p^{m+r_0}}.$$

Then, if we put $g_1 = g_0^c$, we get

$$g_1ag_1^{-1} = a^{1+ck_0p^{n-2m-r_0}}v^{p^{m-r_0}cj_0} = a^{1+ck_0p^{n-2m-r_0}}v^{p^{m-r_0}},$$

and

$$g_1vg_1^{-1} = v.$$

Thus the proof of the theorem is completed.

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