

Kinematic Control of Redundant Manipulator Aiming at Obstacle Avoidance

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Abstract: A method to solve the kinematic position control problem of the 7-DOF redundant manipulator is presented in this paper. One joint angle and one orientation of the end-effector are used as the redundant parameters. The analytical solutions of the inverse kinematics are derived as functions of the redundant parameters. The redundant parameters are modified so that the links get out of the obstacle while the end-effector always keeps the specified position. The interference between the obstacle and the link are tested using lines representing the link and the edge of the obstacle. The distances between the center of the obstacle and those lines are used to test the position of the link: inside of the obstacle and apart enough from the obstacle. The simulation results show the successful avoidance of the obstacle.

Keywords: redundant manipulator, inverse kinematics, redundant parameter, obstacle avoidance, line distance

1. Introduction

The redundant manipulator has functional advantages like the possibility of obstacle and singularity avoidance while it has a difficulty that it has infinite number of joint solutions for a workspace. Most of the kinematic control method of the redundant manipulator is velocity control based on the Jacobian pseud-inverse matrix¹⁾. The velocity control method using the Jacobian matrix has been successfully applied to many tasks, e.g. manipulability but has disadvantages: the computational expensiveness and numerical error accumulation²⁾. The kinematic position control that does not need to calculate the Jacobian pseud-inverse matrix has also been proposed. The method that uses the specified joint angle as the parameter to be decided later based on the predetermined reference point is an example²⁾. The method that decide the elbow position and orientation based on the geometry³⁾ and the idea that uses the rotation angle of the elbow triangle as the parameter⁴⁾ are also examples.

In this research, one or two redundant parameters including the specified joint angle are used to solve the redundant kinematics and the obstacle avoidance of the manipulator.

2. Kinematic definition of 7-DOF redundant manipulator

A 7-DOF manipulator shown in Fig. 1 with $l_2=91$ mm, $l_4=223$ mm, $l_6=196$ mm, and $l_H=250$ mm, is analyzed in the following. The manipulator has 7 revolute joints $J_1, J_2, J_3, J_4, J_5, J_6$, and J_7 . A reference frame $\Sigma_0: x_0, y_0, z_0$ is defined at the base (shoulder) of the manipulator and a frame $\Sigma_H: x_H, y_H, z_H$ is attached to the end-effector. Joint i has the frame $\Sigma_i: x_i, y_i, z_i$ and the joint angle θ_i represents the joint rotation around the z_i axis. The geometric and

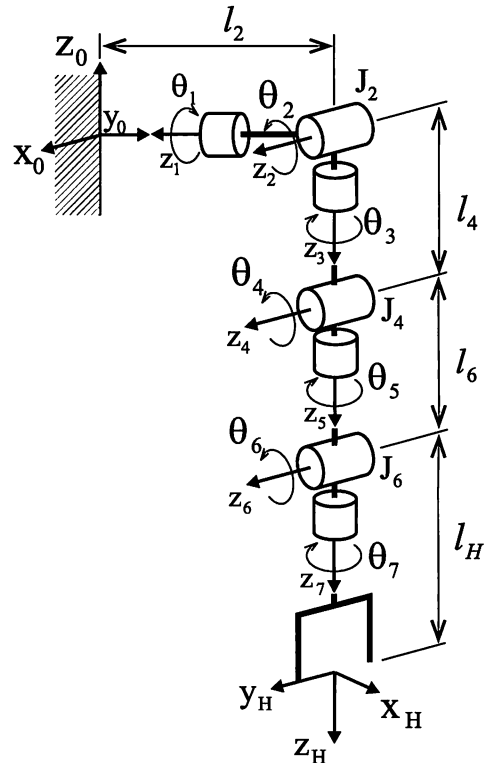


Fig. 1 7-DOF redundant manipulator

kinematic relations between two successive joints are described using the Modified Denavit-Hartenberg notation. The frame Σ_{i-1} is transformed into the frame Σ_i after the translation a_i along the x -axis, the twist α_i around the x -axis, the translation d_i along the z -axis, and the twist θ_i around the z -axis. The modified Denavit-Hartenberg parameters of this manipulator are shown in Table 1.

The frame Σ_i is related to the frame Σ_k by using the 4×4

Table 1 Modified Denavit–Hartenberg parameters

| link i | a_i | α_i | d_i | θ_i |
|----------|-------|-------------|--------|-----------------------|
| 1 | 0 | 90° | $-l_2$ | $\theta_1 - 90^\circ$ |
| 2 | 0 | -90° | 0 | $\theta_2 + 90^\circ$ |
| 3 | 0 | 90° | l_4 | θ_3 |
| 4 | 0 | -90° | 0 | θ_4 |
| 5 | 0 | 90° | l_6 | θ_5 |
| 6 | 0 | -90° | 0 | θ_6 |
| 7 | 0 | 90° | 0 | θ_7 |
| H | 0 | 0° | l_H | 0° |

homogeneous transformation matrix:

$${}^kT_i = \begin{pmatrix} {}^k\mathbf{n}_i & {}^k\mathbf{t}_i & {}^k\mathbf{b}_i & {}^k\mathbf{p}_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

where ${}^k\mathbf{n}_i$, ${}^k\mathbf{t}_i$ and ${}^k\mathbf{b}_i$ are the unit vectors showing the directions of x, y and z axes of the frame Σ_i with respect to the frame Σ_k . The vector ${}^k\mathbf{p}_i$ is the origin of the frame Σ_i with respect to the frame Σ_k . The entries of the matrix kT_i are constructed by using the parameters listed in Table 1.

3. Inverse kinematic solutions with a redundant joint parameter

The task space that this manipulator must satisfy is supposed to be expressed as the 4×4 homogeneous matrix W :

$$W = \begin{pmatrix} \mathbf{n} & \mathbf{t} & \mathbf{b} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

where \mathbf{n} , \mathbf{t} and \mathbf{b} are the unit vectors representing the directions of the x, y and z axes of the end-effector frame Σ_H respectively. The vector \mathbf{p} in Eq. (2) represents the designated objective position of the origin of Σ_H with respect to the base frame Σ_0 . If the task space requirement is satisfied by the end-effector then the following equation is described:

$$W = {}^0T_H = {}^0T_7 T_H \quad (3)$$

Since the transformation 7T_H includes only the translation l_H along the z-axis, the origin of the frame Σ_7 , i.e. ${}^0\mathbf{p}_7 = (r_x, r_y, r_z)^T$ is calculated irrespectively of other joint angles²⁾.

$${}^0\mathbf{p}_7 = \mathbf{p} - l_H \mathbf{b} \quad (4)$$

The task space at the joint 7, W_7 , if defined, can be described in term of the ${}^0\mathbf{p}_7$:

$$W_7 = \begin{pmatrix} \mathbf{n} & \mathbf{t} & \mathbf{b} & {}^0\mathbf{p}_7 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

The joint angles that satisfy the matrix equation

$$W_7 = {}^0T_7 \quad (6)$$

are the solutions of the inverse kinematics. The joint angle θ_3 is selected as a redundant parameter to be fixed later based on the obstacle avoidance condition. Then the following inverse kinematics problem is almost same as that of the non-redundant manipulator³⁾. Multiplying both sides of Eq. (6) by ${}^0T_1^{-1}$ yields:

$${}^0T_1^{-1} \cdot W_7 = {}^1T_7 \quad (7)$$

Equating entries in the fourth column on each side of the Eq. (7), the equations including only the joint angle θ_1 on

left sides are obtained:

$$r_x s_1 - r_z c_1 = l_6(c_2 c_4 - s_2 c_3 s_4) + l_4 c_2 \quad (8)$$

$$r_x c_1 + r_z s_1 = l_6 s_3 s_4 \quad (9)$$

$$l_2 - r_y = -l_6(c_2 c_3 s_4 + s_2 c_4) - l_4 s_2 \quad (10)$$

where s_k and c_k denote $\sin \theta_k$ and $\cos \theta_k$ respectively. The sum of squares of Eq. (8), (9) and (10) results in the equation that contain θ_4 as an unknown parameter:

$$r_x^2 + (l_2 - r_y)^2 + r_z^2 = l_4^2 + l_6^2 + 2l_4 l_6 c_4 \quad (11)$$

Solving Eq. (11), θ_4 is obtained independently of other joint angles.

$$\theta_4 = \pm \tan^{-1} \left(\frac{\sqrt{(2l_4 l_6)^2 - \{r_x^2 + (l_2 - r_y)^2 + r_z^2 - l_4^2 - l_6^2\}}}{r_x^2 + (l_2 - r_y)^2 + r_z^2 - l_4^2 - l_6^2} \right) \quad (12)$$

Analyzing Eq. (9) and (10) gives θ_1 and θ_2 respectively as the functions of the redundant parameter θ_3 when θ_4 is already known:

$$\theta_1 = \tan^{-1} \frac{r_z}{r_x} \pm \tan^{-1} \frac{\sqrt{r_x^2 + r_z^2 - (l_6 s_4)^2 \cdot s_3^2}}{l_6 s_4 \cdot s_3} \quad (13)$$

$$\theta_2 = \tan^{-1} \frac{l_6 c_4 + l_4}{l_6 s_4 \cdot c_3} \mp \tan^{-1} \frac{\sqrt{(l_6 s_4)^2 \cdot c_3^2 + (l_6 c_4 + l_4)^2 - (r_y - l_2)^2}}{r_y - l_2} \quad (14)$$

Multiplying both sides of Eq. (6) by ${}^0T_4^{-1}$, matrix equation to calculate θ_5 , θ_6 and θ_7 is obtained:

$${}^0T_4^{-1} \cdot W_7 = {}^4T_7 \quad (15)$$

Equating elements in the third column on each side of the Eq. (15) gives the relations between the joint variables θ_1 , θ_2 , θ_3 , θ_4 and the unknown variables θ_5 , θ_6 :

$$f_{13}(\theta_1, \theta_2, \theta_3, \theta_4) = c_5 s_6 \quad (16)$$

$$f_{23}(\theta_1, \theta_2, \theta_3, \theta_4) = -c_6 \quad (17)$$

$$f_{33}(\theta_1, \theta_2, \theta_3, \theta_4) = s_5 s_6 \quad (18)$$

where $f_{ij}(\theta_1, \theta_2, \theta_3, \theta_4)$ is a element in row i , column j of the left side of Eq. (15). Dividing Eq. (18) by Eq. (16) when $\sin \theta_6 \neq 0$ yields θ_5 as a function of the redundant parameter θ_3 providing θ_1 , θ_2 and θ_4 are known:

$$\theta_5 = \tan^{-1} \frac{f_{33}(\theta_1, \theta_2, \theta_3, \theta_4)}{f_{13}(\theta_1, \theta_2, \theta_3, \theta_4)} \quad (19)$$

Eq. (17) gives θ_6 as a function of the θ_3 :

$$\theta_6 = \pm \tan^{-1} \frac{\sqrt{f_{13}(\theta_1, \theta_2, \theta_3, \theta_4)^2 + f_{33}(\theta_1, \theta_2, \theta_3, \theta_4)^2}}{f_{23}(\theta_1, \theta_2, \theta_3, \theta_4)} \quad (20)$$

The element in row 2, column 1 and the element in row 2, column 2 of both sides of Eq. (15) are equated to give the equation for θ_7 :

$$f_{21}(\theta_1, \theta_2, \theta_3, \theta_4) = s_6 c_7 \quad (21)$$

$$f_{22}(\theta_1, \theta_2, \theta_3, \theta_4) = -s_6 s_7 \quad (22)$$

Dividing Eq. (22) by Eq. (21) when $\sin \theta_6 \neq 0$ gives θ_7 as a function of θ_3 :

$$\theta_7 = \tan^{-1} \frac{-f_{22}(\theta_1, \theta_2, \theta_3, \theta_4)}{f_{21}(\theta_1, \theta_2, \theta_3, \theta_4)} \quad (23)$$

Changing the value of the redundant joint angle θ_3 from -180° to 180° , the manipulator links make the trajectory shown in Fig. 2. The joint 4 rotates around the line from joint J_2 to joint J_6 . This trajectory implies the possibility of this 7-DOF manipulator to avoid an obstacle to some extent. Fig. 2 shows, however, that this manipulator also has a possibility to avoid an obstacle in wider range.

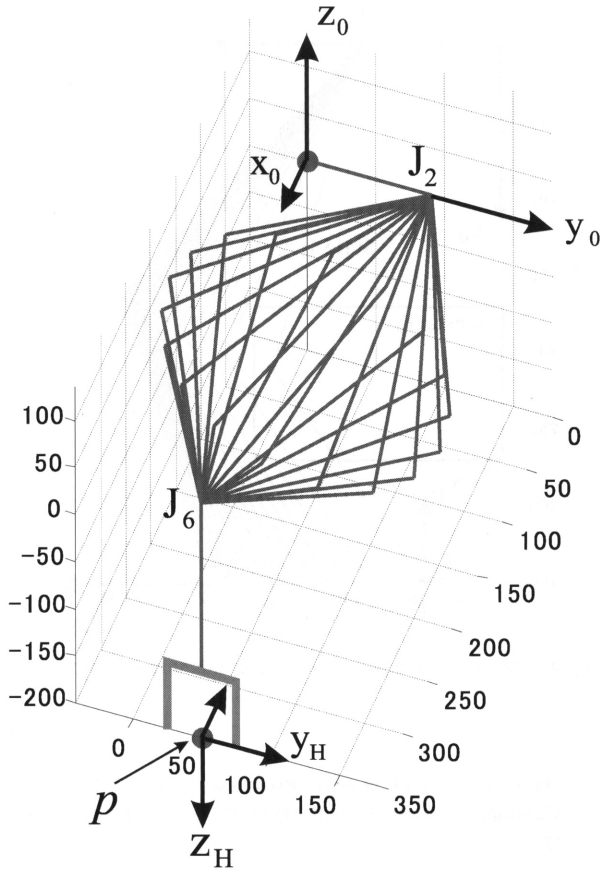


Fig. 2 Trajectory of 7-DOF manipulator changing redundant joint angle θ_3 from -180° to 180°

4. Inverse kinematic with two redundant parameters

The manipulator sometimes has job situation where the strict positioning, especially the control of the orientation of the end-effector is dispensable, e.g. on the way to the destination. If sacrificing the orientation of the end-effector in that situation, an obstacle avoidance of wider range can be expected.

An additional redundancy is considered using the orientation of the end-effector as the redundancy parameter. Let the end-effector can take any orientation keeping y_H-z_H plane parallel to the y_0-z_0 plane while the position of the origin of the end-effector is strictly equal to p . The condition $n = (\pm 1 \ 0 \ 0)^T$ for the unit vector n and $b_x = 0$ for the variable b_x in the unit vector b are necessary to fulfill those orientation of the end-effector. The element b_z of the vector b is used as the redundant parameter. The element b_y and the vector t are decided as functions of the b_z using the condition of unit vector, $|b| = 1$ and orthogonality, $t = b \times n$ respectively. After deciding the unit vector n , t and b as the functions of the redundant parameter b_z ($-1 \leq b_z \leq 1$), the same procedure of Eq. (4) to Eq. (23) are used to calculate the joint angle $\theta_1, \theta_2, \theta_3, \theta_6$ and θ_7 as the function of the another redundant parameter θ_3 . The value of θ_4 can be decided again separately.

5. Simulation: obstacle avoidance using two redundant parameters

The possibility of the obstacle avoidance using these two redundant parameters b_z and θ_3 is examined placing a rectangular parallelepiped obstacle of size $100 \times 160 \times 350$ in the work space of the manipulator. The interference between the link and the obstacle is tested based on the distance between two straight lines, L_m and L_o before the actual joint angles are implemented to joints. The line L_m represents one of the links of the manipulator and the line L_o expresses the edge of the obstacle. As for the link between J_4 and J_6 , any point M_4 on the line L_m corresponding to this link is expressed as:

$$M_4 = {}^0p_4 + t_4 \cdot v_4 \quad (24)$$

where 0p_4 is the position vector in the fourth column of the transformation matrix 0T_4 , t_4 is any constant, v_4 is the unit vector describing the direction of this line L_m . The vector v_4 is expressed as:

$$v_4 = \frac{1}{l_6} ({}^0p_6 - {}^0p_4) \quad (25)$$

The point M on the edge line L_o of the obstacle is

$$M = {}^0p_{O1} + t_o \cdot v_o \quad (26)$$

where ${}^0p_{O1}$ is the position vector of an end of the edge line of the obstacle with respect to the frame Σ_0 , t_o is arbitrary constant, v_o is the unit vector describing the direction of this edge line. Suppose the distance between the point M_m on the line L_m and the point M_o on the line L_o is u_s , i.e. the shortest distance between the line L_m and L_o . The conditions that the line $M_o - M_m$ is perpendicular to v_4 and v_o give the point M_m and M_o :

$$M_m = {}^0p_4 + \frac{\Delta p \cdot v_4 - (\Delta p \cdot v_o) \cdot (v_4 \cdot v_o)}{1 - (v_4 \cdot v_o)^2} v_4 \quad (27)$$

$$M_o = {}^0p_{O1} + \frac{(\Delta p \cdot v_o) \cdot (v_4 \cdot v_o) - \Delta p \cdot v_o}{1 - (v_4 \cdot v_o)^2} v_o \quad (28)$$

where $\Delta p = {}^0p_{O1} - {}^0p_4$.

Let C represent the center of the obstacle, u_m represent the distance between C and M_m , and u_o represent the distance between C and M_o . If $u_m < u_o$, the line L_m that represents the link is inside the obstacle volume. If not so, the line L_m is outside the obstacle and u_s must be greater than R , i.e. the radius of the link. If the obstacle interfere with the manipulator with current configuration, two redundant parameters b_z and θ_3 are changed until u_m exceed u_o in case of $u_m < u_o$, and u_s exceed R in case of $u_m > u_o$. Increasing or decreasing the parameters b_z and θ_3 is decided by evaluating the inclination of u_m in case of $u_m < u_o$, and u_s in case of $u_m > u_o$.

Fig. 3 shows the result of the simulation of the obstacle avoidance of the link between J_4 and J_6 with edge line AB of the obstacle. The calculation of the joint angles to avoid the obstacle starts with $b_z = -1$ and $\theta_3 = -45^\circ$. Iteration stops at $b_z = -0.83$ and $\theta_3 = 0^\circ$ when u_s exceed $R = 50$. Fig. 4 is the top view of the same simulation of Fig. 3. At step 5 with $b_z = -0.96$ and $\theta_3 = -25^\circ$ the line L_m successfully gets out of the obstacle.

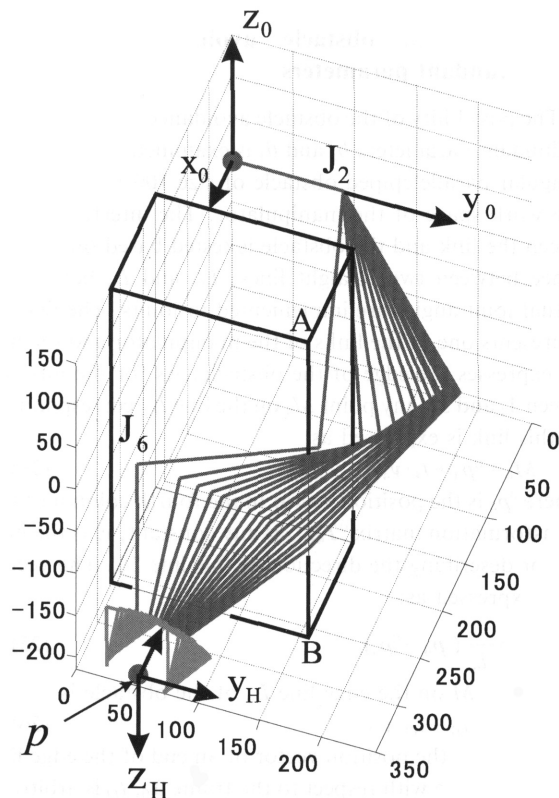


Fig. 3 Obstacle avoidance with the redundant manipulator using two redundant parameters

6. Conclusion

A method to solve the kinematic position control problem of the 7-DOF redundant manipulator is presented in this paper. One joint angle and one orientation of the end-effector are used as the redundant parameters those are decided so that the manipulator link can avoid the obstacle. The interference and the avoidance are tested using the distance and position relation between two lines which represent the link and the edge of the obstacle. The simulation results show the successful avoidance of the obstacle.

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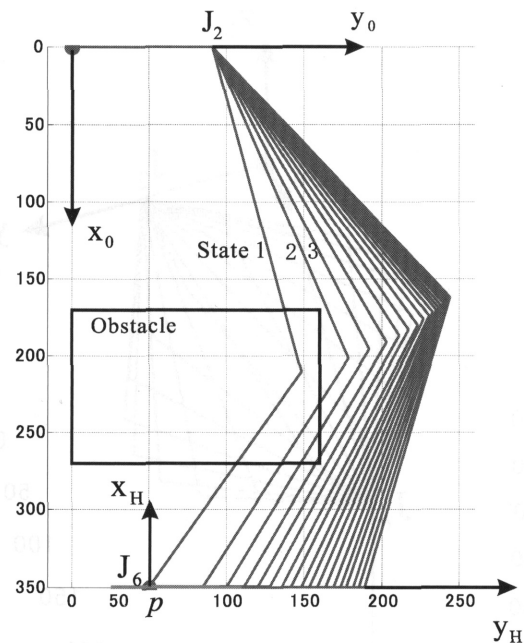


Fig. 4 Obstacle avoidance with the redundant manipulator: top view

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