

A Study on the New Numerical Calculation Method of Dynamic Characteristics and Simulation Model for Internal Combustion Engine Valve Train System —Especially, Cam Lift, Acceleration and Jerk Curves of New Designed Cam Profiles—

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Abstract: Runge-Kutta-Gill's method has been widely used for the dynamic motion numerical calculation of the internal combustion engine valve train system. This paper refers to a new simulation method of the valve train dynamic motion using the transition matrix method. This is a kind of step by step numerical calculation method, making indirect use of Taylor series. As the numerical calculation in this method are made, taking into consideration the constant coefficients in the equation of motion, it has the advantage that the time required for making numerical calculations can be so much reduced. And, to evaluate the new simulation method, we compare the simulated results with the measured data. The numerical calculated results is shown to agree sufficiently with the measured results. Next, to investigate the valve train dynamic characteristics, we change the shape of the cam profile, especially the shape of the cam lift-acceleration (jerk) curve as the numerical calculation conditions, and simulate the dynamic motion of each cam profile using both Runge-Kutta-Gill's method and the transition matrix method in the same equivalent model of the valve train system. The numerical calculated results by the two method almost agree. Also, it is shown that the abnormal behavior of the valve train dynamic motion can be sufficiently reduced. As the conclusion, it is shown that the transition matrix method can be used such a useful tool as Runge-Kutta-Gill's method to numerical calculate the valve train dynamic motion, and that the suitable change of the cam lift-acceleration (jerk) curve of the cam profile can effectively reduce the abnormal behavior of the dynamic motion in the internal combustion engine valve train system from the analytical viewpoints. The validity of this method was confirmed by comparing and examining the measurement results and the simulation numerical calculation.

Key words: Internal Combustion Engine, Valve Train System, Simulation, Numerical Analysis, Modeling/
Runge-Kutta-Gill's Method, Transition Matrix Method, Dynamic Motion, Valve Lift,
Acceleration, Jerk, Jump, Bounce, Cam Profile

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1. Introduction

In recent years, the design of the valve train system is getting one of the most important subjects of the engine design in the demands for the high speed and power. Especially, to cope with the proper engine problems, it is necessary for the internal combustion engine to have not only the good dynamic characteristics but also the sufficient volumetric efficiency in the high speed region^[1]. Therefore, it is important to investigate how the dynamic characteristics of the valve train system change due to the cam lift-acceleration in consideration of the volumetric efficiency.

Besides we suggest that the transition matrix method is useful as a simple and powerful tool of the valve train dynamic simulation like Runge-Kutta-Gill's method [18]. There are many papers on the cam profile and dynamic characteristics of the valve train system [2] – [17], [19] – [22], but most of them refers only to the estimation of the dynamic characteristics [23] – [44]. A. R. Heath [3] has suggested that Multi-Pol cam profile can control the shape of cam lift-acceleration, and that the jerk curve and the asymmetric cam profile between the opening and closing sides can be designed effectively. And, H. Tani [5] has indicated that the maximum and gradient values of the positive acceleration curve have an effect on the dynamic characteristics of the valve train system. Kurisu [6] has suggested that the asymmetric shape of cam lift-acceleration curve has an effect on the improvement of the dynamic characteristics. In this paper, we refer to the better way to get a good dynamic characteristics without reducing volumetric efficiency at the same design conditions of the cam profile [23] – [44].

2. Acceleration Curve Shape and Dynamic Characteristics of Main Cam Profiles

There are various methods of designing the cam profile, but the ones mainly used are Polydyne and Multi-Pol. Polydyne is mainly used in terms of dynamic characteristics of engine valve trains, and Multi-Pole is used as an excellent method in terms of performance. The outline of the design of both cam profiles is described below. Refer to Figure 1.

2.1. Polydyne Cam Profile

The valve lift curve is

$$y = y_{\max} \cdot \left(1 + C_2 \cdot x^2 + C_p \cdot x^p + C_q \cdot x^q + C_r \cdot x^r + C_s \cdot x^5\right) \quad (1)$$

$$y_0 = y_r + A \cdot \frac{dy}{dt} + B \cdot \frac{d^2y}{dt^2} = y_0 = y_r + A \cdot \dot{y} + B \cdot \ddot{y} \quad (2)$$

becomes here,

$$\dot{y} = \frac{dy}{dt} = \frac{d\theta}{dt} \cdot \frac{dy}{d\theta} = \omega \cdot \frac{dy}{d\theta}, \quad \ddot{y} = \omega^2 \cdot \frac{d^2y}{d\theta^2}, \quad x = \left(\frac{\theta}{\theta_r}\right),$$

$$A = \frac{(k_e + k_{sp})}{k_e}, \quad B = \frac{\omega^2 \cdot M}{k_e},$$

next to

y : Cam lift [m], y_{\max} : Maximum cam lift (excluding buffer section) [m], y_0 : Lift height after dynamic characteristic modification [m], C_2, C_p, C_q, C_r, C_s : Coefficient [], θ_r : Half (1/2) of valve opening angle [rad], A : Correction coefficient for static deformation [],

B : Correction coefficient for dynamic deformation [], y_r : Height of shock-absorbing part [m], k_e : Equivalent rigidity of valve train system [Nm/rad], k_{sp} : Spring constant of valve spring [Nm/rad], M : Equivalent mass of valve train system [kg], ω : Cam angular velocity [rad/s]. The boundary condition is

$$y(1) = 0, \quad \frac{dy}{dt}(1) = \dot{y}(1) = 0, \quad \frac{d^2y}{dt^2} = \ddot{y}(1) = 0, \\ \frac{d^3y}{dt^3}(1) = y^{(3)}(1) = \frac{\frac{dy}{dt}}{B} = \frac{\dot{y}_r}{B}, \quad \frac{d^4y}{dt^4}(1) = y^{(4)}(1) = 0 \quad (3)$$

Where: $\frac{dy}{dt} = \dot{y}_r$ is the angular velocity at the buffer. Next,

determine the values of C_2, C_p, C_q, C_r, C_s . Calculated from the boundary conditions and the values of the selected indices p, q, r, s , of C_2, C_p, C_q, C_r, C_s , and substituted into equation (1) to obtain the valve lift curve.

2.2. Multi-Pol Cam Profile

As shown in Figure 1, the valve lift curve is numerical calculated in five sections. The following equation is adopted for each section.

$$y = y_0 + c_1 \cdot \left(\frac{\theta}{\theta_i}\right) + c_2 \cdot \left(\frac{\theta}{\theta_i}\right)^2 + c_3 \cdot \left(\frac{\theta}{\theta_i}\right)^3 + c_4 \cdot \left(\frac{\theta}{\theta_i}\right)^4 + c_5 \cdot \left(\frac{\theta}{\theta_i}\right)^5 \quad (4)$$

Where: θ_i is the cam angle given to each section. Next, the boundary conditions and coefficients shown in Table 1 are applied, and the cam profile is numerical calculated by substituting in equation (4) for each section. This method has a problem of jerk discontinuity in terms of dynamic characteristics. Refer to Table 1, second section.

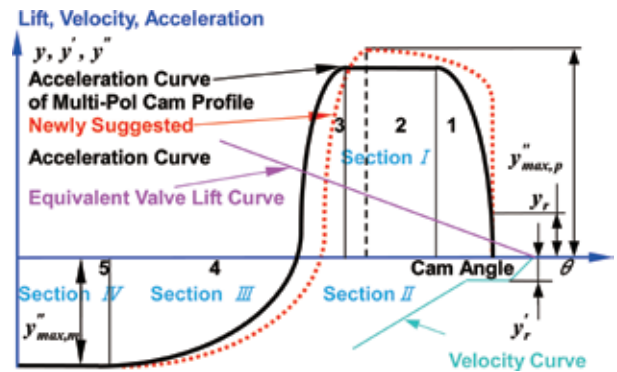


Figure 1 Comparison of Cam Profile [Valve Lift, Velocity and Acceleration] Curves between Multi-Pol and New Suggestion (New Designed)

3. Review of Acceleration Curve of Cam Profile and Dynamic Characteristics in two Typical Method

There are several methods to design cam profile. In this paper, we review two typical methods, that is, Polydyne and Multi-Pol methods because they have been widely used till now. Multi-Pol method is more effective than Polydyne method to make the asymmetric cam lift-acceleration curve in the cam profile design and is useful for the valve train system of a rocker arm type. But it is not so easy to get acceptable dynamic characteristics. In the other hand, Polydyne method is easy to design cam profile and effective to get good dynamic characteristics, but it has usually a higher value of maximum positive acceleration and the cam profile should be symmetric.

4. New Suggestion for Shape of Cam Profile Acceleration Curve

- [1] To make the jerk curve continuous, the second section of Multi-Pol cam profile could be omitted in Figure. 1.
- [2] To consider the dynamic factor, the acceleration continuity is added at the first section.
- [3] And, the lift curve equation is composed of polynomials of the fifth power for the third and fourth sections and polynomials of the fourth power for the first and second sections, respectively, to make the jerk curve continuous.
- [4] The boundary conditions are almost the same as Multi-Pol's but those of the jerk and acceleration curves are different, as shown in Table 2.

Table 1 Boundary Conditions for Multi-Pol Cam Profile [Cam Lift, Velocity, Acceleration and Jerk Curves]

Section	Lift		Velocity		Acceleration		Jerk	
	Start	End	Start	End	Start	End	Start	End
1	y_r	U	$\frac{d y_r}{d t} = \dot{y}_r$	U	0	$\frac{d^2 y_{1-2}}{d t^2} = \ddot{y}_{1-2}$	U	0
2	y_{1-2}	U	$\frac{d y_{1-2}}{d t} = \dot{y}_{1-2}$	U	$\frac{d^2 y}{d t^2} = \ddot{y}_{1-2}$	$\frac{d^2 y_{1-2}}{d t^2} = \ddot{y}_{1-2}$	0	0
3	y_{2-3}	U	$\frac{d y_{2-3}}{d t} = \dot{y}_{2-3}$	U	$\frac{d^2 y}{d t^2} = \ddot{y}_{1-2}$	0	0	$\frac{d^3 y_{3-4}}{d t^3} = \ddot{y}_{3-4}$
4	y_{3-4}	U	$\frac{d y_{3-4}}{d t} = \dot{y}_{3-4}$	U	0	$\frac{d^2 y_{4-5}}{d t^2} = \ddot{y}_{4-5}$	$\frac{d^3 y_{3-4}}{d t^3} = \ddot{y}_{3-4}$	$\frac{d^3 y_{4-5}}{d t^3} = \ddot{y}_{4-5}$
5	y_{4-5}	U	$\frac{d y_{4-5}}{d t} = \dot{y}_{4-5}$	0	$\frac{d^2 y}{d t^2} = \ddot{y}_{4-5}$	U	$\frac{d^3 y_{4-5}}{d t^3} = \ddot{y}_{4-5}$	0

(U : Shows that Boundary Condition is not Given), (0 : Zero)

Table 2 Boundary Conditions for Newly Suggested Cam Profile [Valve Lift, Velocity, Acceleration and Jerk Curves]

Section	Lift		Velocity		Acceleration		Jerk	
	Start	End	Start	End	Start	End	Start	End
I	y_r	U	$\frac{d y_r}{d t} = \dot{y}_r$	U	$\frac{d^2 y_{1-2}}{d t^2} = \ddot{y}_{1-2}^*$	$\frac{d^2 y_{1-2}}{d t^2} = \ddot{y}_{1-2}$	0	U
II	y_{1-2}	U	$\frac{d y_{1-2}}{d t} = \dot{y}_{1-2}$	U	$\frac{d^2 y_{1-2}}{d t^2} = \ddot{y}_{1-2}$	0	0	$\frac{d^3 y_{2-3}}{d t^3} = \ddot{y}_{2-3}$
III	y_{2-3}	U	$\frac{d y_{2-3}}{d t} = \dot{y}_{2-3}$	U	0	$\frac{d^2 y_{3-4}}{d t^2} = \ddot{y}_{3-4}$	$\frac{d^3 y_{2-3}}{d t^3} = \ddot{y}_{2-3}$	$\frac{d^3 y_{3-4}}{d t^3} = \ddot{y}_{3-4}$
IV	y_{3-4}	U	$\frac{d y_{3-4}}{d t} = \dot{y}_{3-4}$	0	$\frac{d^2 y_{3-4}}{d t^2} = \ddot{y}_{3-4}$	U	$\frac{d^3 y_{3-4}}{d t^3} = \ddot{y}_{3-4}$	0

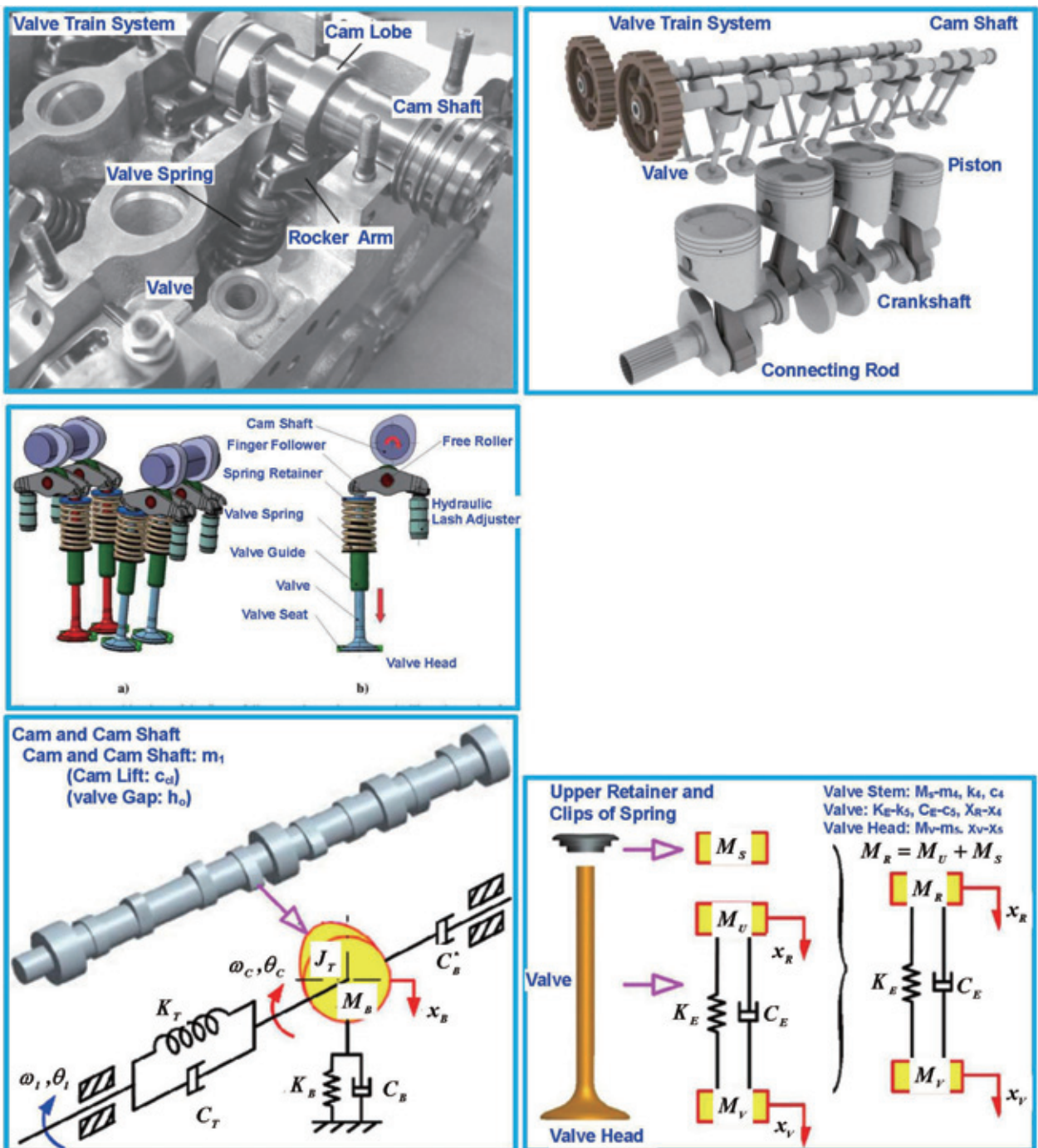
(The value of \ddot{y}_{1-2}^* is approximately 50~95 [%] value of \ddot{y}_{1-2} . And U shows that the boundary condition is not given.)

The section (I , II , III , IV) in **Figure 1** and the section (I , II , III , IV) in **Table 2** are related. In addition, **Figures 1** and **Table 2** show the valve lift, velocity, acceleration, and jerk curves at the start and end of each section.

5. Valve Train Model and New Simulation Method of Valve Train Dynamic Motion

5-1. Multi-degree of Freedom Equivalent Vibration Model of Valve Train System

In the numerical simulation, we adopt an equivalent model of valve train system shown in **Figure 2**.



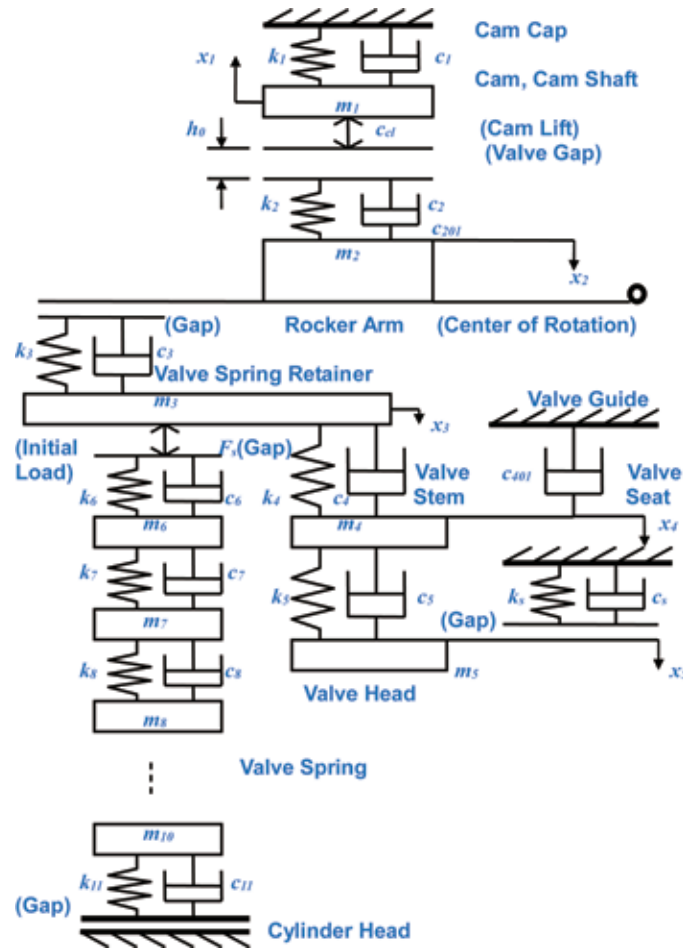


Figure 2 Ten-fold Degree of Freedom Equivalent Vibration Model of Valve Train System

Figure 2 shows the actual valve train system, the 3D CAD model, and the replacement of the cam and camshaft and valve with equivalent models. In addition, a numerical simulation model of the entire valve train system is shown. The validity of this method was confirmed by comparing and examining the measurement results and the numerical calculation simulation.

The main features are as follows:

- [1] The model of valve train system is composed of masses, springs and dampers, and it is assumed to be excited by the displacement of cam profile (cam lift).
- [2] Some elements in this model can be separated from their adjacent elements as shown in Figure 2, in order to analyze such dynamic motions as jump and bounce.
- [3] The friction damping acted between value and valve guide are measured to get accurate data necessary for the simulation.
- [4] There is a stopper between the valve and the valve seat to estimate the impact force of the valve seating.

[5] Sakai's theory is considered in the valve spring model (refer to reference [2]).

[6] Initial force of valve spring (F_s) and initial deflection of valve seat by initial force (δ_{seat}):

(a) F_s is calculated from the initial deflection on the installed condition.

(b) $\delta_{seat} = \frac{F_s}{k_s}$ (k_s : stiffness of valve seat).

5-2. Equation of Motion and Transition Matrix Method

In this section, we derive firstly the equation of motion for each mass and next refer to the simulation method for the dynamic valve train motion by the transition matrix method to compare with Runge-Kutta-Gill's method, which is unnecessary for the review (refer to reference [18]). The equation of motion for each mass is as follows: And the following equations are based on normal conditions which don't excite jump and bounce.

Mass : m_1 (Cam and cam shaft)

$$\begin{aligned} m_1 \cdot \frac{d^2 x_1}{dt^2} + (c_1 + c_2) \cdot \frac{dx_1}{dt} - c_2 \cdot \frac{dx_2}{dt} + (k_1 + k_2) \cdot x_1 - k_2 \cdot x_2 &= -c_2 \cdot \frac{dx_{cl}}{dt} - k_2 \cdot (x_{cl} - h_0) \\ = \\ m_1 \cdot \ddot{x}_1 + (c_1 + c_2) \cdot \dot{x}_1 - c_2 \cdot \dot{x}_2 + (k_1 + k_2) \cdot x_1 - k_2 \cdot x_2 &= -c_2 \cdot \dot{x}_{cl} - k_2 \cdot (x_{cl} - h_0) \end{aligned} \quad (5)$$

Mass : m_2 (Cam and cam shaft)

$$\begin{aligned} m_2 \cdot \frac{d^2 x_2}{dt^2} - c_2 \cdot \frac{dx_1}{dt} + (c_2 + c_3 + c_{201}) \cdot \frac{dx_2}{dt} - c_3 \cdot \frac{dx_3}{dt} - k_2 \cdot x_1 + (k_2 + k_3) \cdot x_2 - k_3 \cdot x_3 &= c_2 \cdot x_{cl} + k_2 \cdot (x_{cl} - h_0) \\ = \\ m_2 \cdot \ddot{x}_2 - c_2 \cdot \dot{x}_1 + (c_2 + c_3 + c_{201}) \cdot \dot{x}_2 - c_3 \cdot \dot{x}_3 - k_2 \cdot x_1 + (k_2 + k_3) \cdot x_2 - k_3 \cdot x_3 &= c_2 \cdot x_{cl} + k_2 \cdot (x_{cl} - h_0) \end{aligned} \quad (6)$$

Mass : m_3 (Cam and cam shaft)

$$\begin{aligned} m_3 \cdot \frac{d^2 x_3}{dt^2} - c_3 \cdot \frac{dx_2}{dt} + (c_3 + c_4 + c_6) \cdot \frac{dx_3}{dt} - c_4 \cdot \frac{dx_4}{dt} - k_3 \cdot x_2 + (k_3 + k_4 + k_6) \cdot x_3 - k_4 \cdot x_4 &= -F_s \\ = \\ m_3 \cdot \ddot{x}_3 - c_3 \cdot \dot{x}_2 + (c_3 + c_4 + c_6) \cdot \dot{x}_3 - c_4 \cdot \dot{x}_4 - k_3 \cdot x_2 + (k_3 + k_4 + k_6) \cdot x_3 - k_4 \cdot x_4 &= -F_s \end{aligned} \quad (7)$$

Mass : m_4 (Cam and cam shaft)

$$\begin{aligned} m_4 \cdot \frac{d^2 x_4}{dt^2} - c_4 \cdot \frac{dx_3}{dt} + (c_4 + c_5 + c_{401}) \cdot \frac{dx_4}{dt} - c_5 \cdot \frac{dx_5}{dt} - k_4 \cdot x_3 + (k_4 + k_5) \cdot x_4 - k_5 \cdot x_5 &= 0 \\ = \\ m_4 \cdot \ddot{x}_4 - c_4 \cdot \dot{x}_3 + (c_4 + c_5 + c_{401}) \cdot \dot{x}_4 - c_5 \cdot \dot{x}_5 - k_4 \cdot x_3 + (k_4 + k_5) \cdot x_4 - k_5 \cdot x_5 &= 0 \end{aligned} \quad (8)$$

Mass : m_5 (Cam and cam shaft)

$$\begin{aligned} m_5 \cdot \frac{d^2 x_5}{dt^2} - c_5 \cdot \frac{dx_4}{dt} + (c_5 + c_s) \cdot \frac{dx_5}{dt} - k_5 \cdot x_4 + (k_5 + k_s) \cdot x_5 &= k_s \cdot \delta_{seat} \\ = \\ m_5 \cdot \ddot{x}_5 - c_5 \cdot \dot{x}_4 + (c_5 + c_s) \cdot \dot{x}_5 - k_5 \cdot x_4 + (k_5 + k_s) \cdot x_5 &= k_s \cdot \delta_{seat} \end{aligned} \quad (9)$$

Mass : m_6 (Cam and cam shaft)

$$\begin{aligned} m_6 \cdot \frac{d^2 x_6}{dt^2} + c_6 \cdot \left(\frac{dx_6}{dt} - \frac{dx_3}{dt} \right) + c_7 \cdot \left(\frac{dx_6}{dt} - \frac{dx_7}{dt} \right) + k_6 \cdot (x_6 - x_3) + k_7 \cdot (x_6 - x_7) &= 0 \\ = \\ m_6 \cdot \ddot{x}_6 + c_6 \cdot (\dot{x}_6 - \dot{x}_3) + c_7 \cdot (\dot{x}_6 - \dot{x}_7) + k_6 \cdot (x_6 - x_3) + k_7 \cdot (x_6 - x_7) &= 0 \end{aligned} \quad (10)$$

Mass : m_{10} (Cam and cam shaft)

$$\begin{aligned} m_{10} \cdot \frac{d^2 x_{10}}{dt^2} + c_{10} \cdot \left(\frac{dx_{10}}{dt} - \frac{dx_9}{dt} \right) + c_{11} \cdot \frac{dx_{10}}{dt} + k_{10} \cdot (x_{10} - x_9) + k_{11} \cdot x_{10} &= 0 \\ = \\ m_{10} \cdot \ddot{x}_{10} + c_{10} \cdot (\dot{x}_{10} - \dot{x}_9) + c_{11} \cdot \dot{x}_{10} + k_{10} \cdot (x_{10} - x_9) + k_{11} \cdot x_{10} &= 0 \end{aligned} \quad (11)$$

The equations (5) to (11) can be expressed in matrices as follows.

$$\begin{aligned} [M] \cdot \left[\frac{d^2 x}{dt^2} \right] + [C] \cdot \left[\frac{dx}{dt} \right] + [K] \cdot [x] &= [F] \\ = \\ [M] \cdot [\ddot{x}] + [C] \cdot [\dot{x}] + [K] \cdot [x] &= [F] \end{aligned} \quad (12)$$

where,

Mass Matrix

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{10} \end{bmatrix}$$

Damping matrix

$$[C] = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_2 & c_2 + c_3 + c_{201} & -c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_3 & c_3 + c_4 + c_6 & -c_4 & 0 & -c_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_4 & c_4 + c_5 + c_s & -c_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_5 & c_5 + c_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_6 & 0 & 0 & c_6 + c_7 & -c_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_7 & c_7 + c_8 & -c_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_8 & c_8 + c_9 & -c_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_9 & c_9 + c_{10} & -c_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{10} & c_{10} + c_{11} \end{bmatrix}$$

Stiffness matrix

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 + k_6 & -k_4 & 0 & -k_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 + k_s & -k_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_5 & k_5 + k_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_6 & 0 & 0 & k_6 + k_7 & -k_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_7 & k_7 + k_8 & -k_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_8 & k_8 + k_9 & -k_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_9 & k_9 + k_{10} & -k_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{10} & k_{10} + k_{11} \end{bmatrix}$$

Component of forced column vector

$$[F] = \left[-k_2 \cdot (c_{cl} - h_0) - c_2 \cdot \frac{d x_{cl}}{d t}, k_2 \cdot (x_{cl} - h_0) + c_2 \cdot \frac{d x_{cl}}{d t} - F_s, 0, k_s \cdot \delta_{seat}, 0, 0, 0, 0, 0 \right]^T,$$

=

$$[F] = \left[-k_2 \cdot (c_{cl} - h_0) - c_2 \cdot \dot{x}_{cl}, k_2 \cdot (x_{cl} - h_0) + c_2 \cdot \dot{x}_{cl} - F_s, 0, k_s \cdot \delta_{seat}, 0, 0, 0, 0, 0 \right]^T,$$

Component of acceleration column vector

$$\left[\frac{d^2 x}{d t^2} \right] = \left[\frac{d^2 x_1}{d t^2}, \frac{d^2 x_2}{d t^2}, \frac{d^2 x_3}{d t^2}, \frac{d^2 x_4}{d t^2}, \frac{d^2 x_5}{d t^2}, \frac{d^2 x_6}{d t^2}, \frac{d^2 x_7}{d t^2}, \frac{d^2 x_8}{d t^2}, \frac{d^2 x_9}{d t^2}, \frac{d^2 x_{10}}{d t^2} \right]^T$$

$$[\ddot{x}] = [\ddot{x}_1, \ddot{x}_2, \ddot{x}_3, \ddot{x}_4, \ddot{x}_5, \ddot{x}_6, \ddot{x}_7, \ddot{x}_8, \ddot{x}_9, \ddot{x}_{10}]^T$$

Component of velocity column vector

$$\left[\frac{d x}{d t} \right] = \left[\frac{d x_1}{d t}, \frac{d x_2}{d t}, \frac{d x_3}{d t}, \frac{d x_4}{d t}, \frac{d x_5}{d t}, \frac{d x_6}{d t}, \frac{d x_7}{d t}, \frac{d x_8}{d t}, \frac{d x_9}{d t}, \frac{d x_{10}}{d t} \right]^T$$

$$[\dot{x}] = [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5, \dot{x}_6, \dot{x}_7, \dot{x}_8, \dot{x}_9, \dot{x}_{10}]^T$$

Component of displacement column vector

$$[x] = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}]^T$$

T : Transposition

The transition matrix method, which is a kind of step by step numerical calculation methods making indirect use of Taylor series, is used to solve equation (12). We have already referred to the method in detail by reference [1]. The following equation is finally obtained on referring to our paper [1], considering up to the fourth derived function.

$$\begin{aligned} \begin{bmatrix} \frac{x}{d t} \\ \frac{d x}{d t} \end{bmatrix}_{k+1} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} x \\ \frac{d x}{d t} \end{bmatrix}_k + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \cdot \begin{bmatrix} F \\ \frac{d F}{d t} \end{bmatrix}_k + \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \cdot \begin{bmatrix} \frac{d^2 F}{d t^2} \\ \frac{d^2 F}{d t^2} \end{bmatrix}_k \\ &= \\ \begin{bmatrix} x \\ \dot{x} \end{bmatrix}_{k+1} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \end{bmatrix}_k + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \cdot \begin{bmatrix} F \\ \dot{F} \end{bmatrix}_k + \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \cdot \begin{bmatrix} \ddot{F} \\ \ddot{F} \end{bmatrix}_k \end{aligned} \quad (14)$$

where A_{ij} , B_{ij} and C_{ii} ($i=1,2, j=1,2$), which constitute the transition matrices, are partial matrices of 10×10 composed of the step size which satisfies the stabilizing condition, and the various factors constituting the equivalent vibration system shown in Figure 1. If the particulars of the equivalent vibration system, step size and excitation torque are determined, the partial matrices A_{ij} , B_{ij} , C_{ii} ($i=1,2, j=1,2$) and the exciting terms F , $\frac{d F}{d t} = \dot{F}$, $\frac{d^2 F}{d t^2} = \ddot{F}$ in equation (14) will be known. And, x_{k+1} and $\frac{d x_{k+1}}{d t} = \dot{x}_{k+1}$ will be obtained when the initial values of x_k and $\frac{d x}{d t} = \dot{x}_k$ are given so that the calculations can be made continuously.

5-3. Equation of Motion under Abnormal Dynamic Conditions

For example, if it is assumed that the jump happens at mass 3 ($x_3 > x_2$), the equations are as follows, in the stead of equations (6) and (7).

$$\begin{aligned} m_2 \cdot \frac{d^2 x_2}{d t^2} - c_2 \cdot \frac{d x_1}{d t} + (c_2 + c_{201}) \cdot \frac{d x_2}{d t} - k_2 \cdot x_1 + k_2 \cdot x_2 &= c_2 \cdot c_{cl} + k_2 \cdot (c_{cl} - h_0) \\ &= \\ m_2 \cdot \ddot{x}_2 - c_2 \cdot \dot{x}_1 + (c_2 + c_{201}) \cdot \dot{x}_2 - k_2 \cdot x_1 + k_2 \cdot x_2 &= c_2 \cdot c_{cl} + k_2 \cdot (c_{cl} - h_0) \end{aligned} \quad (15)$$

$$\begin{aligned} m_3 \cdot \frac{d^2 x_3}{d t^2} + (c_4 + c_6) \cdot \frac{d x_3}{d t} - c_4 \cdot \frac{d x_4}{d t} + (k_4 + k_6) \cdot x_3 - k_4 \cdot x_4 &= -F_s \\ &= \\ m_3 \cdot \ddot{x}_3 + (c_4 + c_6) \cdot \dot{x}_3 - c_4 \cdot \dot{x}_4 + (k_4 + k_6) \cdot x_3 - k_4 \cdot x_4 &= -F_s \end{aligned} \quad (16)$$

Therefore, the damping and stiffness matrices have to be changed as follows.

Damping matrix

$$[C] = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_2 & c_2 + c_{201} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_4 + c_6 & -c_4 & 0 & -c_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_4 & c_4 + c_5 + c_s & -c_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_5 & c_5 + c_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_6 & 0 & 0 & c_6 + c_7 & -c_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_7 & c_7 + c_8 & -c_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_8 & c_8 + c_9 & -c_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_9 & c_9 + c_{10} & -c_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{10} & c_{10} + c_{11} \end{bmatrix}$$

Stiffness matrix

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_4 + k_6 & -k_4 & 0 & -k_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 + k_s & -k_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_5 & k_5 + k_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_6 & 0 & 0 & k_6 + k_7 & -k_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_7 & k_7 + k_8 & -k_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_8 & k_8 + k_9 & -k_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_9 & k_9 + k_{10} & -k_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{10} & k_{10} + k_{11} & 0 \end{bmatrix}$$

The displacement of each mass is calculated on condition that the values of $[C]$ and $[K]$ changes every calculating step on the basis of the dynamic characteristics.

6. Measured Value Motion and Rocker Arm Force

The main specifications of the test gasoline engine are shown on the Table 3. The type of valve train system is rocker arm type, finger follower. We adopt the experimental data offered by some engine maker, and review them from the following points of view.

6-1. Valve Motion and Decision of Bounce

From the data of the valve lift (displacement) measured by the sensor, we can easily find whether the bounce on the valve seat occurs or not (refer to Figure 3).

Figure 3 shows the relationship between the valve lift and cam angle at each engine speed as an example of experimental results. (It's zooming of between $\frac{5}{4}\pi$ and $\frac{7}{4}\pi$ [rad] at 5000 to 7000 [r/min]). The valve lift was measured using a leather gap sensor, and the cam angle was measured using a rotary encoder. As the engine speed increases, so does the valve lift between $\frac{5}{4}\pi$ and $\frac{6}{4}\pi$ [rad] at 5000 to 7000 [r/min] (Bounce). As the engine speed increases, so does the bounce that occurs on the valve. This is due to the valve spring, mass and inertia effect of the cam and camshaft.

6-2. Rocker Arm Force and Decision of Jump

If the jump occurs between the rocker arm and the cam

Table 3 Main Specification of the Test Gasoline Engine

Engine Type	In-line, 4 Cylinders, Gasoline Engine	Automobile Use
Piston Displacement	2.00×10^3 [m ³]	←
Type of Valve Train System	Rocker arm Type	Finger Follower
Maximum Lift	9.4 [mm]	at Valve

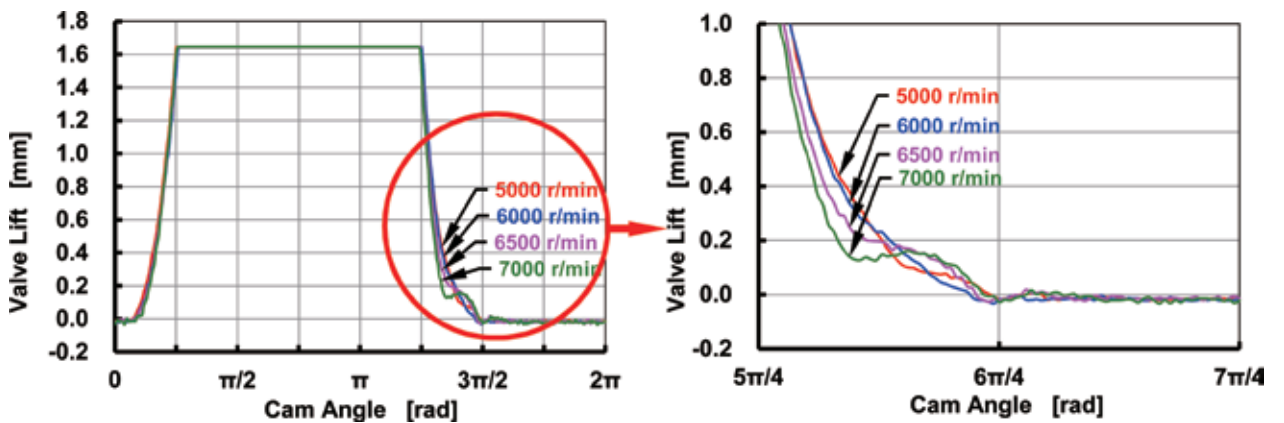


Figure 3 Experimental Results of Original Valve Motions [Valve Lift], [Zooming of between $5\pi/4$ and $7\pi/4$ [rad] at 5000 to 7000 [r/min]]

shaft or between the rocker arm and the valve, or if the bounce occurs from the valve seat, this rocker arm force should be nearly zero. Therefore, if the experimental data of the rocker arm force and the valve displacement are checked the occurrence of the jump or the bounce can be clearly found (refer to Figure 4).

Figure 4 shows an example of the results of measuring the load acting on the rocker arm using a strain gauge and a dynamic strain gauge. (It's zooming of between π and 2π [rad], 2π and 3π [rad] at 5000 to 7000 [r/min]). Since the rocker arm has a complicated shape, a calibration curve was created in advance to calculate the load. As the engine speed increases, so does the load acting on the valve between π and 2π [rad], 2π and 3π [rad] at 5000 to 7000 [r/min] (jump). As the engine speed increases, the load generated on the rocker arm also increases, and the amount of jump also increases. This is due to the mass and inertia effect of the rocker arm.

6-3. Occurrences of Jump and Bounce of Test Engine

From Figures 3 and 4, we can find the abnormal dynamic behavior of the valve train system. Firstly, the small bounce occurs at 5000 [r/min] and the pretty big

bounce occurs from 6500 [r/min] to 7000 [r/min]. Secondly, the jump phenomena occur more than 5000 [r/min].

7. Simulation Results by Runge-Kutta-Gill's Method and New Numerical Calculation Method (Transition Matrix Method)

The simulation results of the valve motion and the rocker arm force using the Runge-Kutta-Gill's method and the Transition matrix method are shown on Figures 5, 7, and Figures 6, 8, respectively. In the simulations the experimental values of the damping coefficient c are adopted, that is, $c = 19.4$ to 24.6 . On the bounce phenomena, their simulation results agree with the measured data. On the jump phenomena, the shapes of the force's curves obtained by their simulations are somewhat different from those obtained by the measurement, but the revolutions of the occurrence are almost the same as the measured ones. One more point to mention is that the numerical calculation results of the valve motion and the rocker arm force by Runge-Kutta-Gill's method agree well with those by the new numerical calculation method (transition matrix method). Therefore, it is assured that their numerical

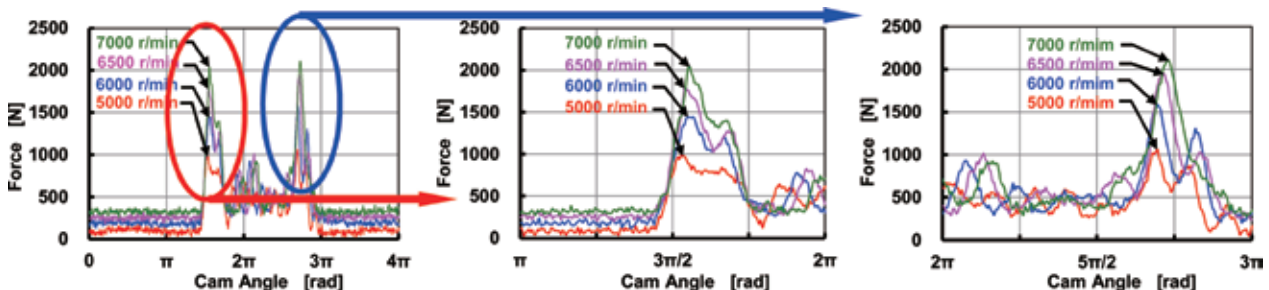


Figure 4 Experimental Results of Original Rocker Arm Loads [Zooming of between π and 2π [rad], 2π and 3π [rad] at 5000 to 7000 [r/min]]

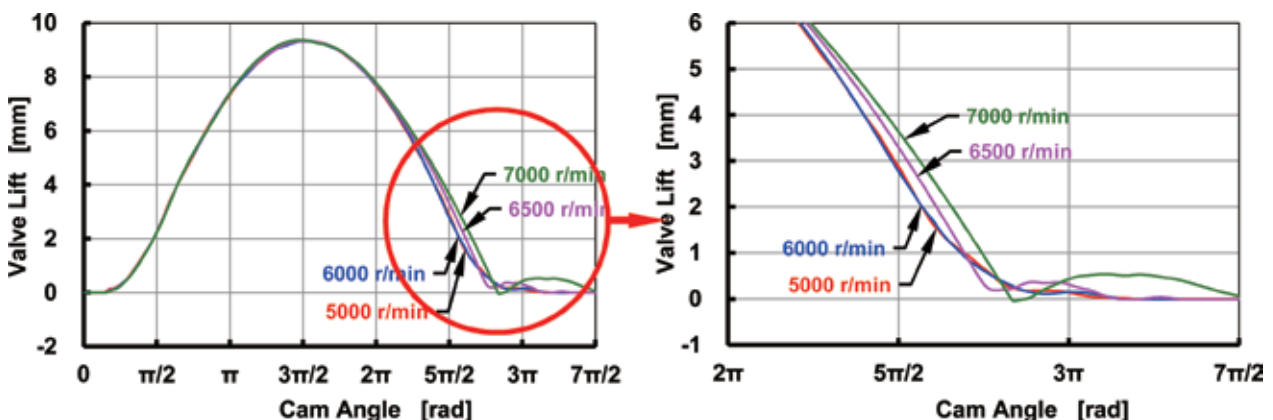


Figure 5 Simulation Results of Original Valve Motions [Valve Lift] by Runge-Kutta-Gill's Method [Zooming of between 2π and $7\pi/2$ [rad] at 5000 to 7000 [r/min]]

calculation methods can simulate the dynamic characteristics of the valve train system. The difference of the rocker arm force between the measured and simulated results arises from the insufficiency of the hydraulic lash adjuster's

modeling. But, judging from the dynamic characteristic points of view, the valve train model and their simulation methods are shown to be available for the numerical calculation of the valve train system.

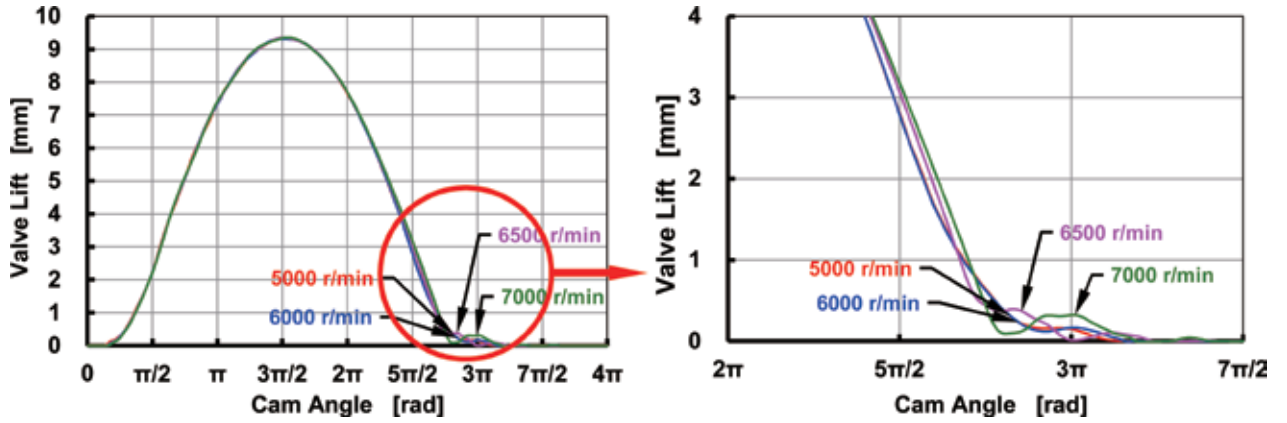


Figure 6 Simulation Results of Original Valve Motions [Valve Lift] by New Numerical Calculation Method (Transition Matrix Method) [Zooming of between 2π and $7\pi/2$ [rad] at 5000 to 7000 [r/min]]

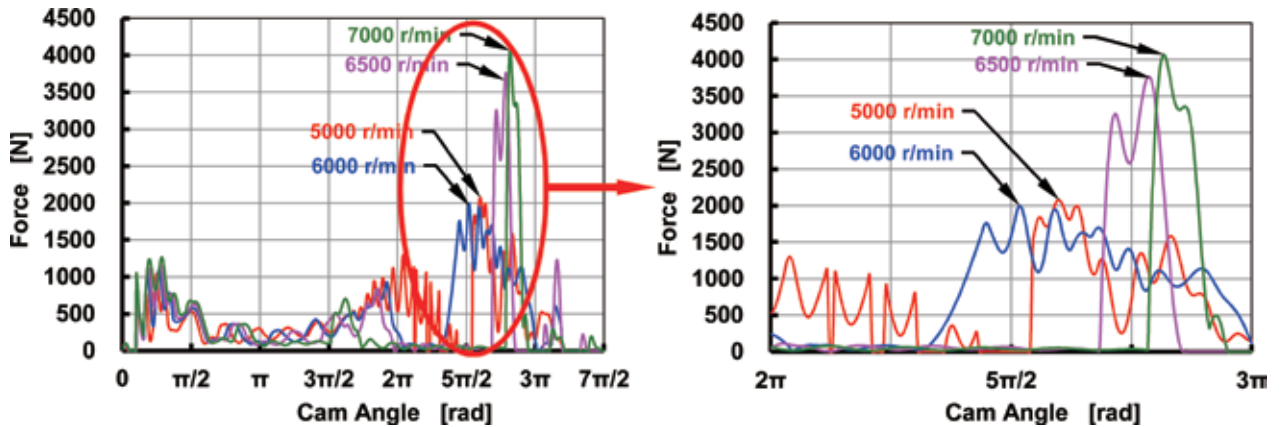


Figure 7 Simulation Results of Original Rocker Arm Loads by Runge-Kutta-Gill's Method [Zooming of between 2π and 3π [rad] at 5000 to 7000 [r/min]]

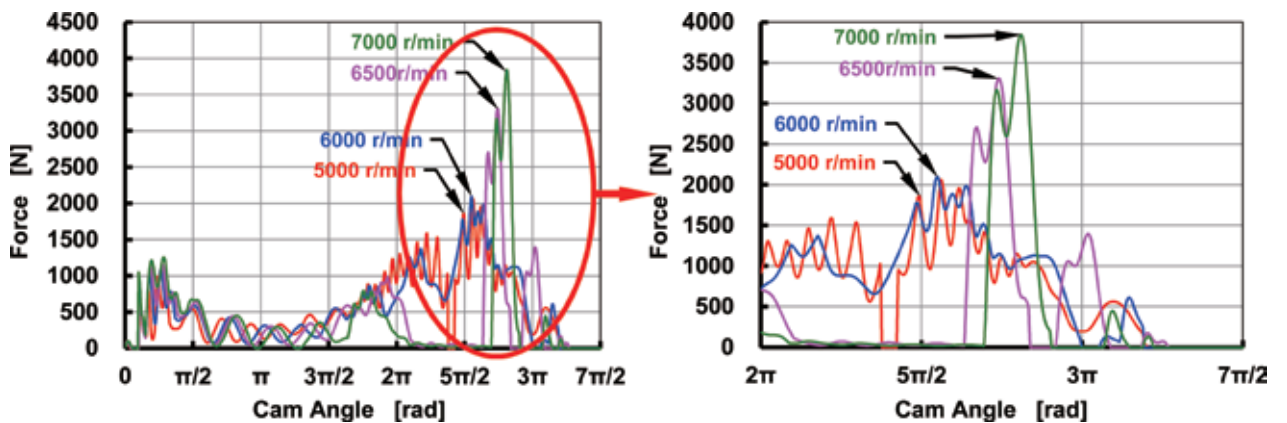


Figure 8 Simulation Results of Original Rocker Arm Loads by New Numerical Calculation Method (Transition Matrix Method) [Zooming of between 2π and $7\pi/2$ [rad] at 5000 to 7000 [r/min]]

8. Investigation of Dynamic Changes due to Difference Cam Profiles

8-1. Design Conditions of Cam Profile

The design of cam profile is carried out on the basis of new suggestions in section 4. The cam profile should be designed so as to have the maximum area ratio and the good dynamic characteristics at the high engine speed under the given conditions. Table 4 shows the design conditions by which three kinds of cam profiles are designed to have the maximum criteria. In case of Polydyne cam profile, it's not so easy to limit maximum positive acceleration value, so Polydyne cam profile is designed without limit value of maximum positive acceleration.

8-2. Comparison of Lift, Acceleration and Jerk Curves of Designed Cam Profiles

The lift, acceleration and jerk curves of the designed cam profiles are shown on Figures 9, 10 and 11. Figure 9 shows the equivalent valve lift curves. The reason why Polydyne cam profile have a little bit bigger area ratio is originated by no limit of positive acceleration value at the design stage. So, it is shown that there is no significant difference among the valve lift curves. But, it is shown that there are some differences on the acceleration and jerk curves in Figures 10 and 11, respectively. Polydyne cam profile has its continuity in the acceleration and jerk curves. The new design cam profile has almost the same tendency as Polydyne. Multi-Pol cam profile has its discontinuity in

Table 4 Design Conditions of Cam Profile [Comparison of Polydyne, Multi-Pol and New Design Cam Profile]

Term	Unit	Polydyne	Multi-Pol	New Design
Cam Duration	[deg]	144	144	144
Maximum Cam Lift	[mm]	9.4	9.4	9.4
Maximum Positive Acceleration	[mm/deg ²]	-----	0.015	0.015
Maximum Negative Acceleration	[mm/deg ²]	0.008	0.008	0.008
Ramp Height (Closing Side)	[mm]	0.250	0.265	0.265
Ramp Velocity	[mm/deg]	0.11	0.11	0.11

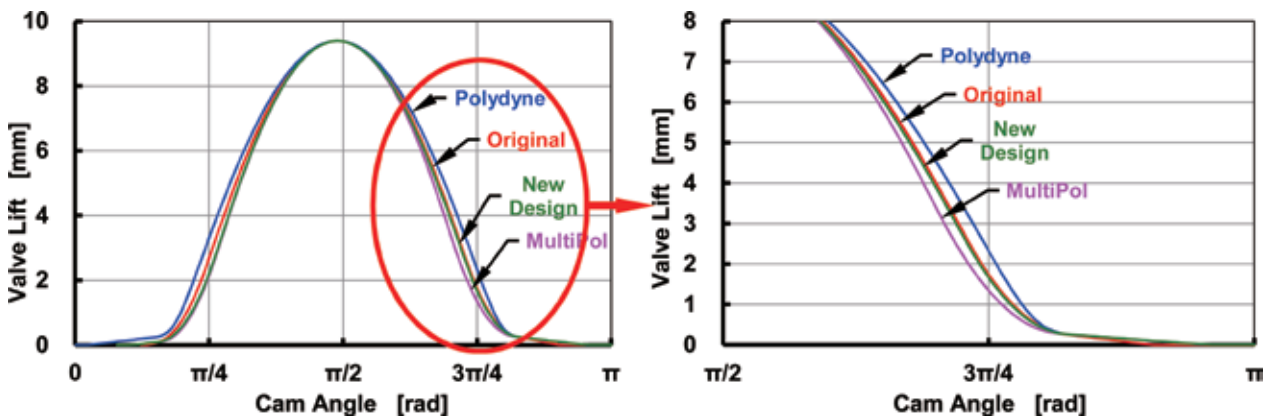


Figure 9 Comparison of Equivalent Valve Lift Curves [Comparison of Valve Lift Curves of Polydyne, Original, New Designed and Multi-Pol], [Zooming of between $\pi/2$ and π [rad]]

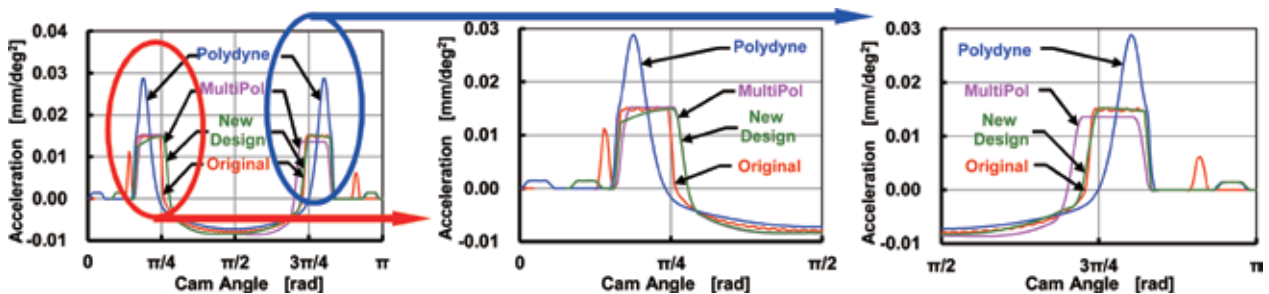


Figure 10 Comparison of Acceleration Curves [Comparison of Valve Lift Curves of Polydyne, Original, New Designed and Multi-Pol], [Zooming of between 0 and $\pi/2$ [rad], $\pi/2$ and π [rad]]

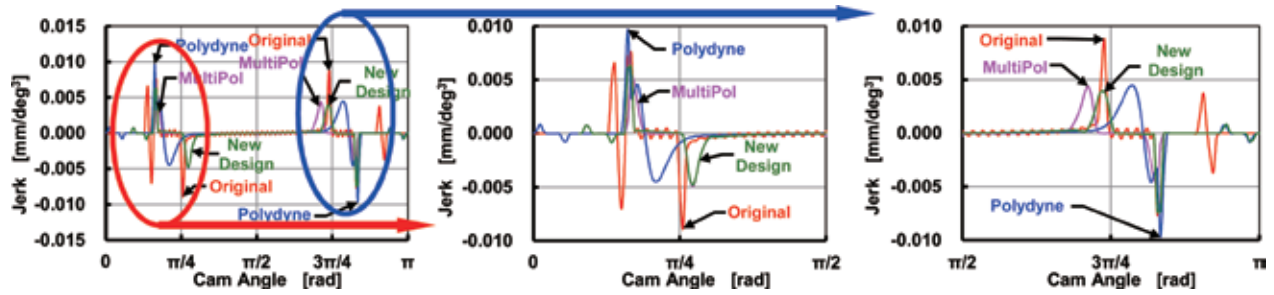


Figure 11 Comparison of Jerk Curves [Comparison of Valve Lift Curves of Polydyne, Original, New Designed and Multi-Pol], [Zooming of between 0 and $\pi/2$ [rad], $\pi/2$ and π [rad] at 5000 to 7000 [r/min]]

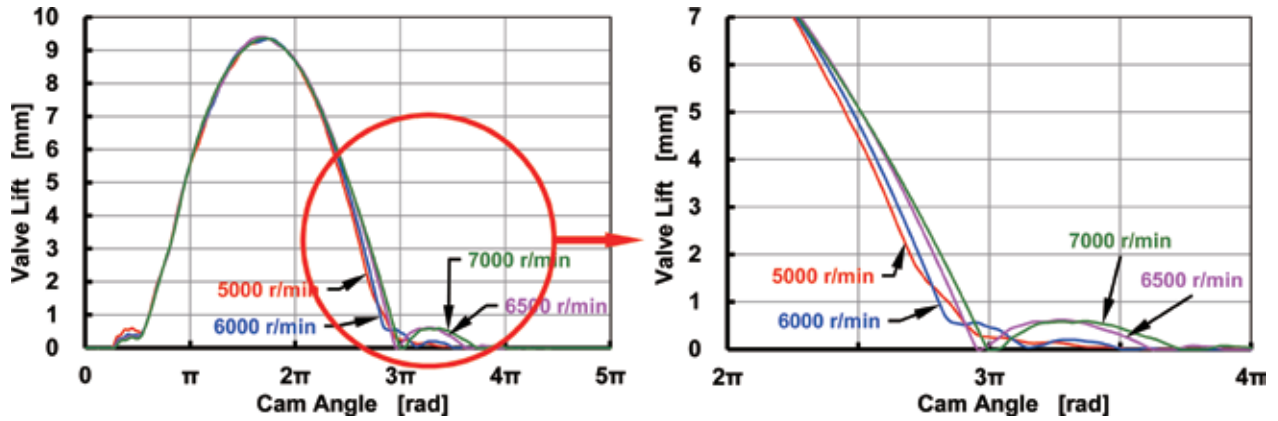


Figure 12 Simulation Results of Multi-Pol Valve Motions [Valve Lift] by Runge-Kutta-Gill's Method [Zooming of between 2π and 4π [rad] at 5000 to 7000 [r/min]]

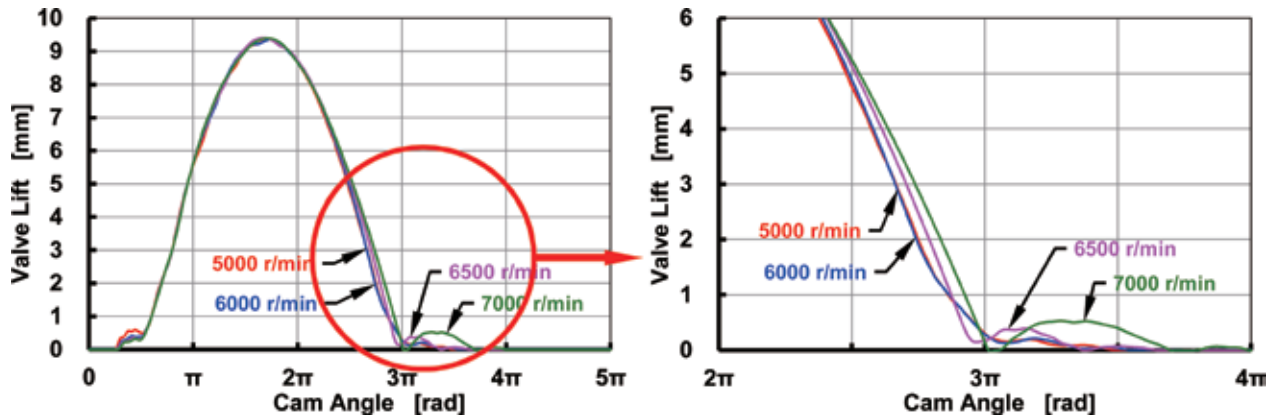


Figure 13 Simulation Results of New Design Valve Motions [Valve Lift] by Runge-Kutta-Gill's Method [Zooming of between 2π and 4π [rad] at 5000 to 7000 [r/min]]

the acceleration and jerk curves.

8-3. Simulation Results

The simulated results are shown in Figures 12, 13, and Figures. 14, 15 using Runge-Kutta-Gill's method and the new numerical calculation method (transition matrix method), respectively. We simulated also Polydyne cam profile by the same methods. And, from Figures. 12 to 15,

it is shown that the simulated results by two methods almost agree. The simulated results are also shown on Table 5 using Runge-Kutta-Gill's method and the new numerical calculation method (transition matrix method). As shown on Table 5, Polydyne cam profile has good dynamic characteristics, but also has some difficulties in adjusting its maximum positive acceleration value. This point becomes a problem in the rocker arm type of valve train system,

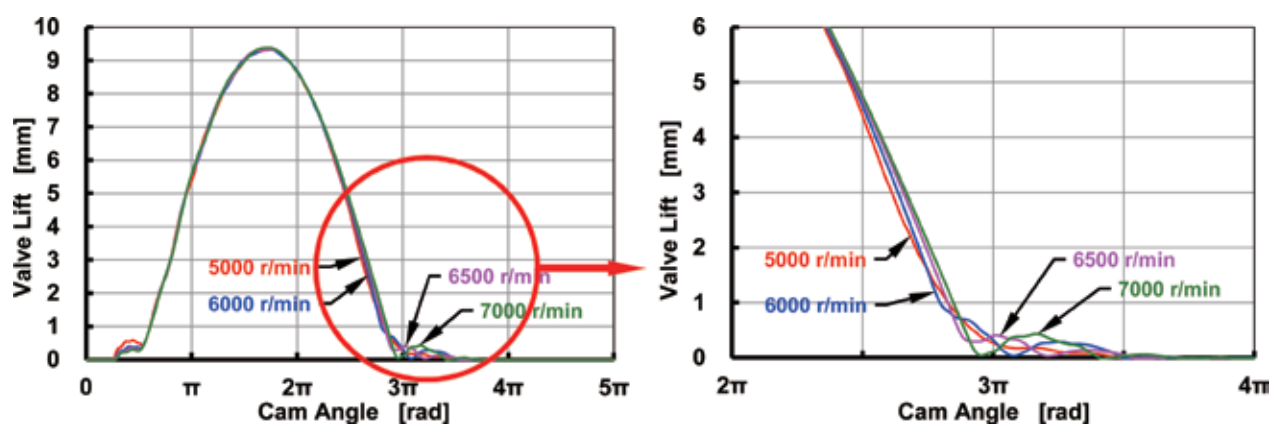


Figure 14 Simulation Results of Multi-Pol Valve Motions [Valve Lift] by New Numerical Calculation Method (Transition Matrix Method) [Zooming of between 2π and 4π [rad] at 5000 to 7000 [r/min]]

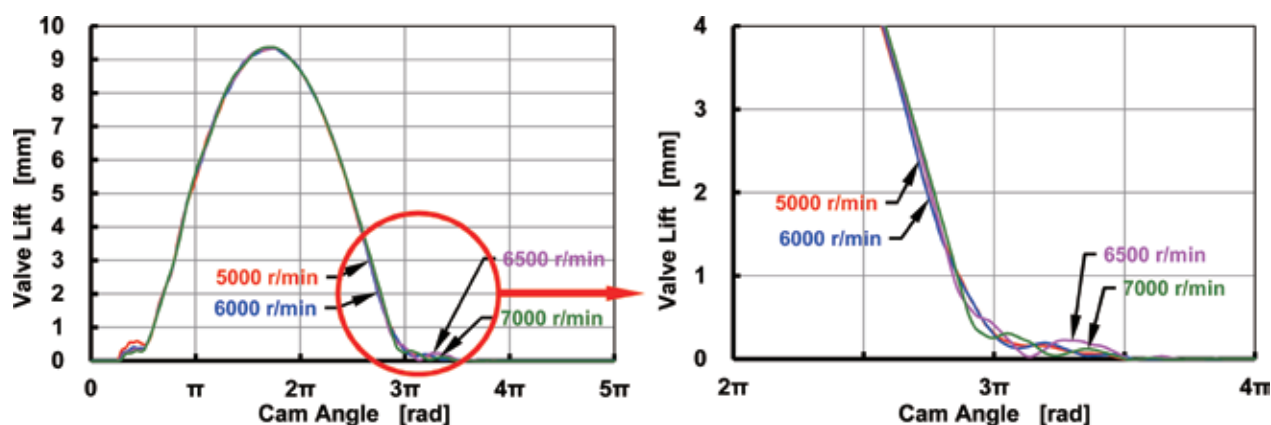


Figure 15 Simulation Results of New Design Valve Motions [Valve Lift] by New Numerical Calculation Method (Transition Matrix Method) [Zooming of between 2π and 4π [rad] at 5000 to 7000 [r/min]]

Table 5 Numerical Simulated Results by Two Method [Comparison of Runge-Kutta-Gill's Method [R.K.G.M.] and Transition Matrix Method [T.M.M.]]

Cam Profile Type	Engine Speed [r/min]	Bounce [mm] (R.K.G.M.*/T.M.M.**)	Average
Polydyne	5000	0.00/0.00	0.00
	6000	0.22/0.20	0.21
	6500	0.22/0.28	0.25
	7000	0.73/0.43	0.58
Multi-Pol	5000	0.00/0.00	0.00
	6000	0.20/0.29	0.25
	6500	0.58/0.41	0.50
	7000	0.57/0.43	0.50
New Design	5000	0.00/0.00	0.00
	6000	0.21/0.19	0.20
	6500	0.39/0.23	0.31
	7000	0.53/0.30	0.42

*: Runge-Kutta-Gill's Method,

**: Transition Matrix Method

especially, on the machining process of the cam lobe. Multi-Pol cam profile can be easily designed but its dynamic characteristics are poor by the discontinuity of the acceleration and jerk curves. Therefore, if the acceleration and jerk curves could be design so as to make easily its curve shape like Polydyne's, there could be some possibility to meet optimum design point just as new design cam profile.

8-4. Comparison of Simulated Results between Runge-Kutta-Gill's Method and New Numerical Calculation Method (Transition Matrix Method).

As shown in Figures 12 to 15, it is assured that the two methods can simulate the valve train dynamics if a good model of the valve train system is adopted and the step size is properly determined. Besides, the new numerical calculation method (transition matrix method) is simple method and an useful tool for the simulation of the valve train system such as Runge-Kutta-Gill's method.

9. Conclusions

In this paper, we have as far discussed Runge-Kutta-Gill's method and the new numerical calculation method (transition matrix method) to simulate the valve train dynamics. And, we compared the simulated results on three kinds of cam profiles in order to get better dynamic characteristics without reducing volumetric efficiency at the same design conditions. The following conclusions are obtained.

- [1] Authors have developed the valve train dynamic simulation program using the new numerical calculation method (transition matrix method). The transition matrix method is shown to be able to be adopted as a simple and powerful tool of the simulation for the valve train dynamics like Runge-Kutta-Gill's method.
- [2] We have suggested a simple way to design the cam profile in order to make the acceleration and jerk curves continuous for the good dynamics of the valve train system. Especially, the bounce phenomena have been improved if the designer make the shapes of the acceleration and jerk curves on the closing side continuous like Polydyne cam profile.
- [3] The validity of this method was confirmed by comparing and examining the measurement results and the simulation numerical calculation.

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