論文　**Original Paper**

A Study on Rubber Shapes and Dynamic Characteristics of some Torsional Vibration Shear Type Rubber Dampers for High Speed Diesel Engine Crankshaft System

Tomoaki Kodama *1, Yasuhiro Honda *2

Abstract：This study refers to the relationship between dynamic characteristics and rubber shapes of some torsional vibration shear type rubber dampers by torsional stiffness and loss constants, which indicate their dynamic characteristics, against shape factors of rubber. The torsional stiffness in this study, which is called a complex torsional stiffness, consists of a typical torsional stiffness and a damping coefficient. The shape factor is determined as the ratio of free loaded area to loaded area of rubber. Four kinds of test shear type rubber dampers that only rubber shapes are different are used in this study. The test is engine experiment with the shear type rubber damper and exciting torsional vibration experiment. The experiment with shear type rubber damper is a high speed diesel engine test in the rated engine speed range with each test shear type rubber damper attached to the crankshaft system front edge. The frequency set in exciting torsional vibration test was made to be able to generator the frequencies occurred at the crankshaft system of the high speed diesel engine. Changing the shape of a pulley for exciting torsional vibration can change the exciting torsional vibration amplitude. The dynamic characteristics obtained from the experiment results are estimated in consideration of rubber shapes. The relation between amplitude dependence and the shape factor is particularly notified. As a results, the following knowledge can be provided.

[1] The dynamic characteristics of the damper depend on shape factor.
[2] The dependent tendency presents conspicuously with increase of the shape factor.
[3] Rubber damper with smaller shape factor contributes to easily predict dynamic characteristics on the design stage. However, the damper of the small shape factor is hard to satisfy a torsional stiffness to tune to the torsional vibration mode of the crankshaft system of the high speed diesel engine.

Key words：Dynamic Characteristics, Shear Type Rubber Damper, Rubber Shape, Torsional Vibration, Shape Factor, Torsional Stiffness, Damping Coefficient, Area Ratio, Diesel Engine, Crankshaft System, Experiment, Forced Frequency Ratio, Amplitude Ratio

1. Introduction

In recent years, from the viewpoint of energy use in the high efficiency, saving of resources and environmental protection, a high speed diesel engine is reconsidered as the engine being able to work with even wide fuel except the light oil has the advantage in the thermal efficiency higher than a gasoline engine [1)~3)]. On the other hand, we have many problem to solve such as reduction of the weight, reduction of noise, vibration and harshness (NVH) and engine performance improvement. One of the solutions is light weighting of the motion parts as one of the improvement methods of the engine performance. The light weighting of the engine crankshaft contributes to the rise of the engine revolution limit of the high speed diesel engine to increase the horsepower but there is a problem that increase of torsional vibration occurring in the crankshaft makes the crankshaft damage [1)~7)]. As a general reduction countermeasure of the torsional vibration, a shear type rubber torsional vibration damper is attached to a engine crankshaft system. Because the shear type rubber torsional vibration damper has amplitude dependence, frequency dependence, temperature dependence, strain rate dependence and so on, it is difficult to grasp the dynamic characteristic precisely [8)~15)]. Furthermore, the difference of the rubber shape changes the dynamic characteristics even if the rubber of the same material was employed to a rubber damper [16)~25)].

This study referred to the relationship among dynamic characteristics and rubber shapes of some torsional vibration shear type rubber dampers by rigidities and loss constants,

*1 Kokushikan University, Dr. of Engineering, kodama@kokushikan.ac.jp
*2 School of Science and Engineering, Kokushikan University, Ph. D., honda@kokushikan.ac.jp

which indicate dynamic characteristics, against shape factors of rubber [1)~3), 4)~6)]. The torsional stiffness in this study, which is called a complex torsional stiffness, consists of typical stiffness and a damping coefficient. The shape factor means the ratio of free loaded area to loaded area of rubber. In other words, this shape factor can be called area ratio by regarding rubber as incompressibility materials. Four kinds of test shear type rubber dampers that only rubber shapes are different are used in this study. The rubber materials of the test dampers are natural rubber and rubber hardness : Hs = 50. The experiment is engine test with the rubber damper and exciting vibration experiment. The experiment with rubber damper is an engine experiment in the rated engine speed range with each test rubber damper attached to the crankshaft system front end. The frequency set in exciting torsional vibration test was made agree with frequency occurred at the crankshaft system of the diesel engine. Changing the shape of exciting vibration pulleys can change the exciting torsional vibration amplitude [7)~11)]. The dynamic characteristics obtained from the experiment results are estimated in consideration of rubber shapes. The relation between amplitude dependence and the shape factor is particularly notified.

2. Test Shear Type Rubber Dampers and Reduction Control on Torsional Vibration

The combustion pressure in the cylinder of the high speed diesel engine is higher than that of the gasoline engine. As a result, bigger torsional vibrations occurs on the engine crankshaft system by exciting torsional vibration force and crank and piston mechanism. A torsional vibration shear type rubber damper is generally used to reduce the torsional vibration occurring on the crankshaft system of the in-line multi-cylinder high speed diesel engine.

Figure 1 shows the genuine torsional vibration shear type rubber damper for in-line 6-cylinder high speed diesel engine

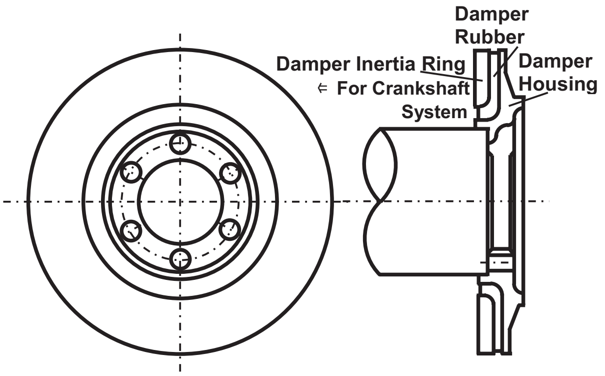

Figure 1 Genuine Shear Type Rubber Damper [Damper STD (S)]

of total displacement volume 6.211 [Litter]. The output of the test engine brake power is 212 [kW] and the brake torque is 402 [Nm]. The major specifications of the test diesel engine are shown in **Table 1**. **Figure 2** shows the torsional vibration amplitude curves occurring in the pulley end of the crankshaft of the test diesel engine without a rubber damper, as the experimental diagram shown in **Figure 3**. The overall value of the amplitude at the resonance point of the sixth vibration of the main torsional vibration component exceeds the permission torsional displacement of the crankshaft system. **Figure 4** shows the torsional vibration amplitude curves of the pulley end of the crankshaft system attached the shear type rubber damper shown in Figure 1. It can be confirmed that this shear type rubber damper absorbing a part of the torsional vibration energy of the crankshaft system reduces the torsional vibrations. However, the torsional vibration reduction may not be controlled as the designer intended because the dynamic characteristics of the torsional vibration shear type rubber damper are influenced on the dependencies of amplitude, frequency, and strain rate temperature.

It is difficult to completely predict the dynamic

Table 1 Main Specifications of the Test Engine

Particulars	Contents
Designed for	High-Speed Diesel Engine
Type for	4-Stroke Cycle, Direct Injection
Number of Cylinders	6 Cylinders
Arrangement	In-Line
Bore and Stroke [m]	0.108-0.133
Total Piston Displacement [m^3]	0.006211
Compression Ratio	18.9
Maximum Brake Output [kW/r/min]	212 / 3200
Maximum Torque [Nm/r/min]	402 / 2000
Firing Order	1 - 4 - 2 - 6 - 3 - 5

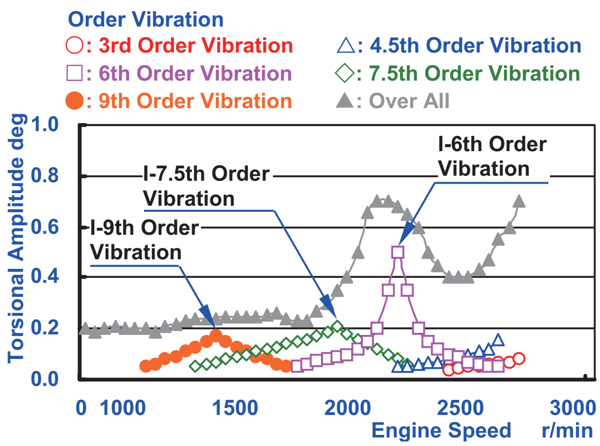

Figure 2 Torsional Vibration Amplitude Curves of Crankshaft Pulley End without Shear Type Rubber Damper [I : Node Vibration]

characteristics with these dependences in the design stage of the shear type rubber damper. Materials of the rubber, shore hardness and rubber shapes make the prediction more difficult. These influence factors can be divided into internal factors and external factors. The combinations of all these factors change the dynamic characteristics.

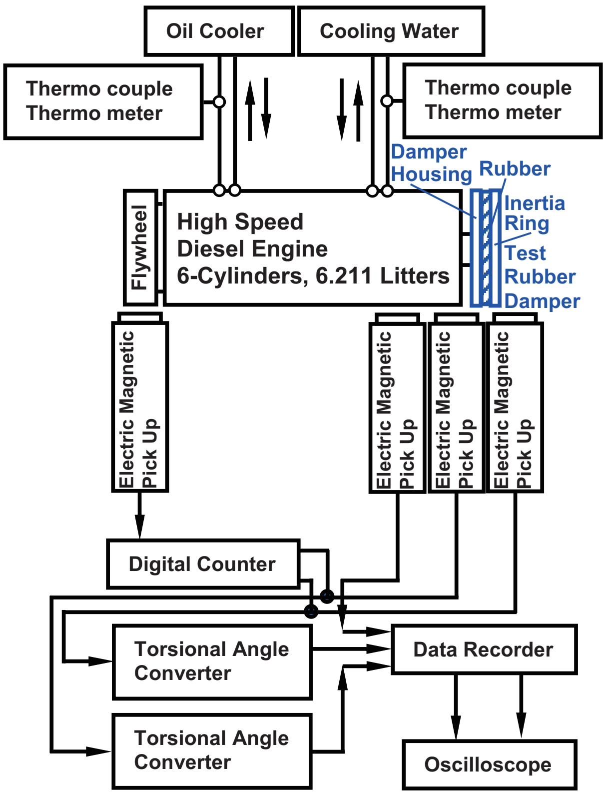

Figure 3 Schematic Diagram of Torsional Vibration Measurement for Engine Experiment

The relations of the dynamic characteristics with amplitude dependence and rubber shapes are investigated in this study to clear the rubber shape and these influence factors.

Figure 5 and **Table 2** indicate the trial manufactured test rubber dampers of which rubber shapes are only different based on the genuine rubber damper shown in Figure 1 and their specifications. The rubbers of all rubber dampers are natural rubber with shore hardness：Hs＝50. Table 2 indicates the inertia moment of damper housing and inertia ring of each test rubber damper. The different values of the moment of inertia can be adjusted to be a constant value by attaching a gear for torsional vibration measurement with the damper housing and the inertia ring respectively.

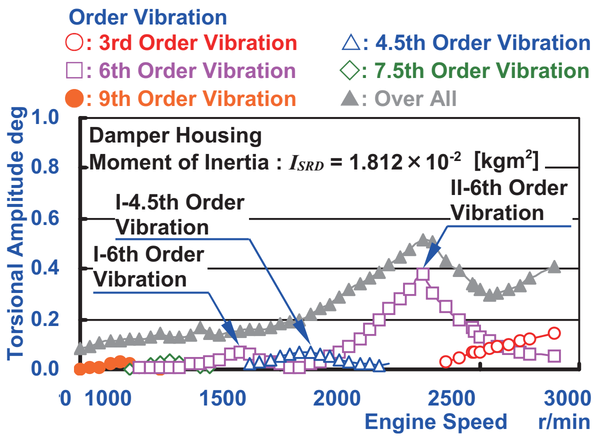

Figure 4 Torsional Vibration Amplitude Curves of Crankshaft Pulley End with Genuine Shear Type Rubber Damper [Rubber Damper STD (S), Moment of Inertia : $I_{SRD} = 1.812 \times 10^{-2}$ [kgm^2], I, II : Node Vibration]

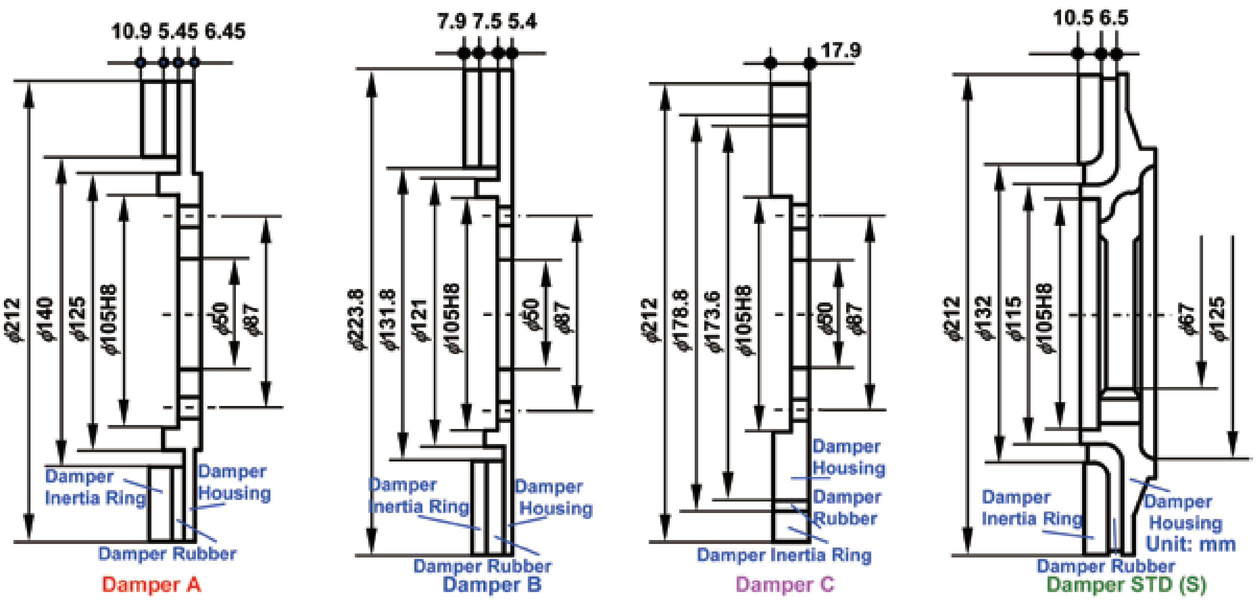

Figure 5 Trial Manufactured Test Shear Type Rubber Dampers with Only Different Rubber Shapes and Damper STD (S)

Table 2 Main Specifications of Test Shear Type Rubber Dampers

Damper Name	Shape Factor of Damper $U_{SRD,d}$ [m³]	Shape Factor	Polar Moment of Area $I_{SRD,x}$ [m⁴]	Volume of Rubber V_{SRD} [m³]
Damper A	2.947×10^{-2}	3.303	1.606×10^{-4}	1.085×10^{-4}
Damper B	2.889×10^{-2}	3.067	2.167×10^{-4}	1.966×10^{-4}
Damper C	2.957×10^{-2}	3.442	7.687×10^{-5}	2.576×10^{-5}
Damper STD (S)	2.948×10^{-2}	3.517	1.970×10^{-4}	1.966×10^{-4}

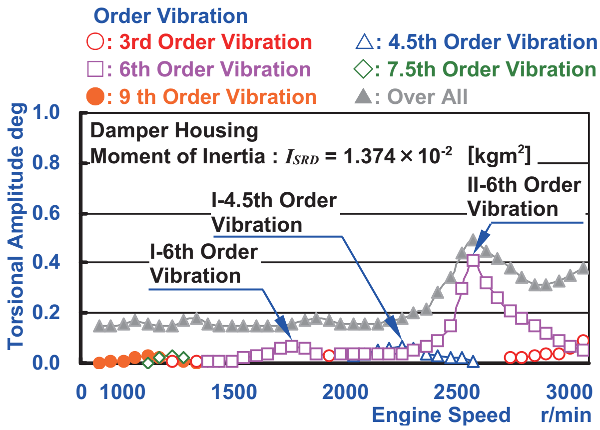

Figure 6 Torsional Vibration Amplitude Curves of Crankshaft Pulley End with Shear Type Rubber Damper A [Moment of Inertia : $I_{SRD}=1.374\times10^{-2}$ [kgm²], I, II : Node Vibration]

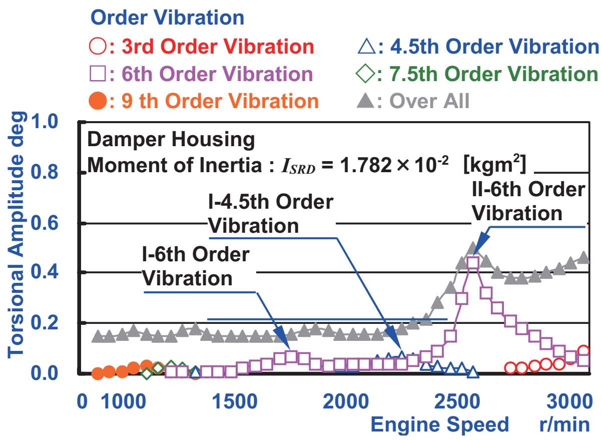

Figure 7 Torsional Vibration Amplitude Curves of Crankshaft Pulley End with Shear Type Rubber Damper C [Moment of Inertia : $I_{SRD}=1.782\times10^{-2}$ [kgm²], I, II : Node Vibration]

Figures 6 and 7 indicate torsional vibration amplitude curves of the test shear type rubber dampers. It is understood that the control on torsional vibration effects of each test shear type rubber damper are different comparing resonance frequencies and resonance amplitudes of the sixth torsional vibration that is a main vibration order component. It can be supposed that the difference of the rubber shape changes the dynamic characteristics of each test shear type rubber dampers. However, it is difficult for reasons of the statement above to solve dynamic characteristics every influence factor of a shear type rubber damper attached to a high speed diesel engine crankshaft system. In this study, the relations of the amplitude dependence of the dynamic characteristic and the rubber shapes are tried to solve based on the results obtained from the torsional vibration experiment by original manufactured exciting torsional vibration machine.

3. Numerical Calculation of the Dynamic Characteristic Value of the Rubber Damper and the Results

It is necessary to dismantle it every torsional vibration order component because the torsional vibration waveform occurring in the crankshaft system of the engine is a compound wave. The value of the dynamic characteristic of the shear type rubber damper can be numerical calculated by the next expressions.

$$I_{SRD}\cdot\frac{d^2\theta_{SRD}}{dt^2}+C_{SRD}\cdot\left(\frac{d\theta_{SRD,d}}{dt}-\frac{d\theta_{SRD,p}}{dt}\right)+K_{SRD}\cdot\left(\theta_{SRD,d}-\theta_{SRD,p}\right)=0$$

$$I_{SRD}\cdot\ddot{\theta}_{SRD}+C_{SRD}\cdot\left(\dot{\theta}_{SRD,d}-\dot{\theta}_{SRD,p}\right)+K_{SRD}\cdot\left(\theta_{SRD,d}-\theta_{SRD,p}\right)=0 \qquad (1)$$

here, $\theta_{SRD,d}$: torsional vibration angular displacement of damper housing [rad], $\theta_{SRD,p}$: torsional vibration angular displacement of the damper inertia ring [rad], I_{SRD} : inertia moment of damper inertia ring [kgm²], C_{SRD} : damping coefficient of damper rubber [Nms/rad], K_{SRD} : torsional stiffness of damper rubber [Nm/rad].

The equation (1) gives the dynamic torsional stiffness and damping coefficient values as follows.

$$K_{SRD}=\frac{I_{SRD}\cdot\omega_{SRD}^2\cdot M_{SRD}\cdot\left(M_{SRD}-\cos\phi_{SRD}\right)}{M_{SRD}^2+1-2\cdot M_{SRD}\cdot\cos\phi_{SRD}} \qquad (2)$$

$$C_{SRD}=\frac{I_{SRD}\cdot\omega_{SRD}\cdot M_{SRD}\cdot\sin\phi_{SRD}}{M_{SRD}^2+1-2\cdot M_{SRD}\cdot\cos\phi_{SRD}} \qquad (3)$$

here, ω_{SRD} : forced frequency [rad/s], M_{SRD} : amplitude ratio $\left(=\frac{\theta_{SRD,do}}{\theta_{SRD,po}}\right)$ [−], $\theta_{SRD,do}$: torsional vibration amplitude of damper housing [rad], $\theta_{SRD,p}$: torsional vibration amplitude of the damper inertia ring [rad]. Furthermore, the absolute value of the torsional stiffness $\left|K_{SRD}^*\right|$ and the ratio of torsional stiffness a_{SRD} are given in the next equations.

$$\left|K_{SRD}^*\right|=\sqrt{K_{SRD}^2+\left(C_{SRD}\cdot\omega_{SRD}\right)^2} \qquad (4)$$

$$a_{SRD} = \frac{K_{SRD}}{\left|K^*_{SRD}\right|} = \frac{\dfrac{K_{SRD}}{I_{SRD}\cdot\omega^2_{SRD}}}{\dfrac{\left|K^*_{SRD}\right|}{I_{SRD}\cdot\omega_{SRD}}} \tag{5}$$

Figures from **8** to **11** indicate the numerical calculated values obtained from equations (4) and (5). These results mean that the relations between the absolute value of the torsional stiffness and the torsional stiffness are linear and the damping torque $C_{SRD}\cdot\omega_{SRD}$ are constant as the a_{SRD} is constant. In other words, the damping coefficient that divided $C_{SRD}\cdot\omega_{SRD}$ by ω_{SRD} shows that it decrease with the increase of the forced frequencies. On the other hand, the torsional stiffness of each damper is changed by the difference of the rubber shape as the a_{SRD} of each rubber damper is different. This fact means that the difference of rubber shape influences on amplitude dependence and frequency dependence of the dynamic characteristics. Choice of the rubber shape that is hard to be affected by each dependency will give a better control on torsional vibration effect, if the dependencies of the dynamic characteristics are not analyzed adequately.

In the same torsional angular displacement, the strains of the rubber are different by rubber shape. Here, a coefficient to indicate a strain conversion factor is introduced. The strain conversion factor is defined as necessary coefficient to convert a torsional angular displacement into strain. The rubber shapes of the test dampers can be divided into the radial direction adhesion type and the axial direction adhesion type as shown in Figure 1 and 4. But the genuine damper of Figure 1 is similar as a combination type of the radial direction adhesion type and the axial direction adhesion type.

[1] The radial direction adhesion type

Area ratio : $U_{SRD,r}$

$$U_{SRD,r} = \frac{r_{SRD,r,2}\cdot\left(1-\mu_{SRD,r}\right)}{2\cdot l_{SRD,r}} \tag{6}$$

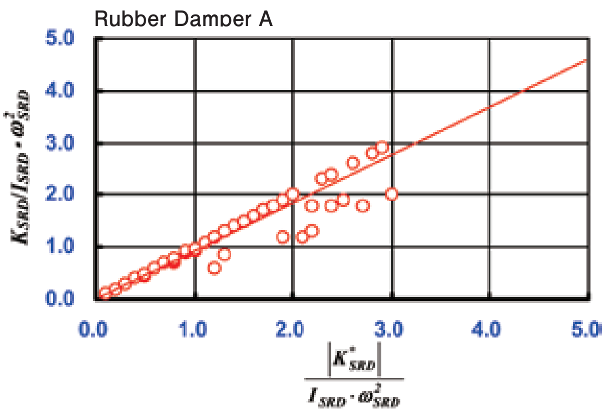

Figure 8 Relationship between Torsional Stiffness and Absolute Value of the Torsional Stiffness [Rubber Damper A]

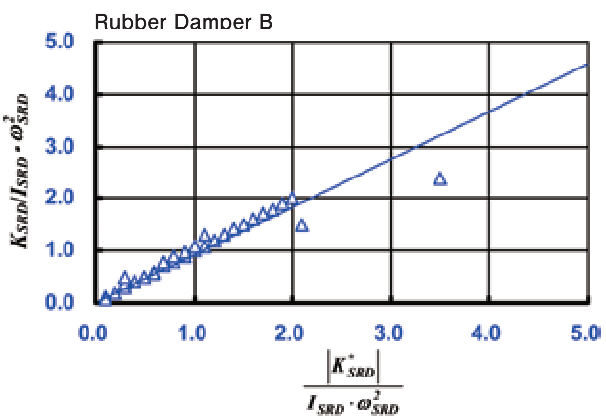

Figure 9 Relationship between Torsional Stiffness and Absolute Value of the Torsional Stiffness [Rubber Damper B]

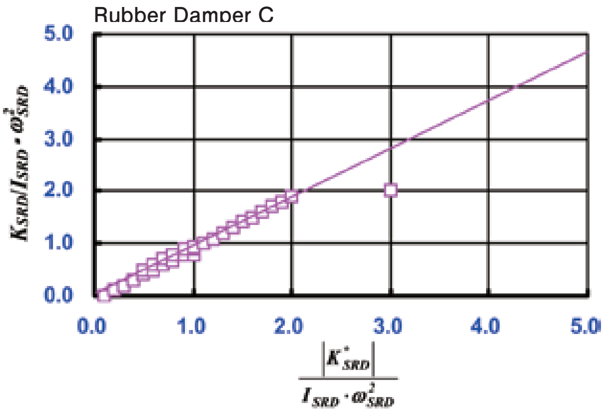

Figure 10 Relationship between Torsional Stiffness and Absolute Value of the Torsional Stiffness [Rubber Damper C]

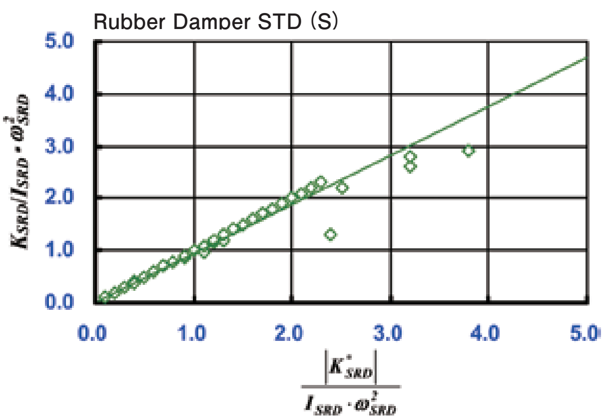

Figure 11 Relationship between Torsional Stiffness and Absolute Value of the Torsional Stiffness [Rubber Damper STD (S)]

Representative position of the strain：$\rho_{SRD,r}$

$$\rho_{SRD,r} = \frac{3\cdot r_{SRD,r,2}\cdot\left(1+\mu_{SRD,r}^{4}\right)}{4\cdot\left(1-\mu_{SRD,r}^{3}\right)}\cdot\theta_{SRD,r,Ro} \qquad (7)$$

Strain：$\gamma_{SRD,r}$

$$\gamma_{SRD,r} = \frac{3\cdot U_{SRD,r}\cdot\left(1+\mu_{SRD,r}\right)\cdot\left(1+\mu_{SRD,r}^{2}\right)}{2\cdot\left(1-\mu_{SRD,r}^{3}\right)}\cdot\theta_{SRD,r,Ro} = U_{SRD,r,\mu}\cdot\theta_{SRD,r,Ro} \qquad (8)$$

Strain conversion factor：$U_{SRD,r,\mu}$

$$U_{SRD,r,\mu} = \frac{3\cdot\left(1+\mu_{SRD,r}\right)\cdot\left(1+\mu_{SRD,r}^{2}\right)}{2\cdot\left(1-\mu_{SRV,r}^{3}\right)} \qquad (9)$$

［2］ The axial direction adhesion type

Area ratio：$U_{SRD,a}$

$$U_{SRD,a} = \frac{l'_{SRD,a}}{r_{SRD,a,2}\cdot\mu_{SRD,a}\cdot\left(1-\mu_{SRD,a}^{2}\right)} \qquad (10)$$

Representative position of the strain：$\rho_{SRD,a}$

$$\rho_{SRD,a} = \gamma_{SRD,a,2}\cdot\sqrt{\frac{\left(1-\mu_{SRD,a}^{3}\right)}{3\cdot\left(1-\mu_{SRD,a}\right)}} \qquad (11)$$

Strain：$\gamma_{SRD,a}$

$$\gamma_{SRD,a} = \frac{1}{1-\mu_{SRD,a}\cdot\sqrt{\dfrac{3\cdot\left(1-\mu\right)}{1-\mu_{SRD,a}^{3}}}}\cdot\theta_{SRD,a,Ro} = U_{SRD,a,\mu}\cdot\theta_{SRD,a,Ro} \qquad (12)$$

Strain conversion factor：$U_{SRD,a,\mu}$

$$U_{SRD,a,\mu} = \frac{1}{1-\mu_{SRD,a}\cdot\sqrt{\dfrac{3\cdot\left(1-\mu_{SRD,r}\right)}{1-\mu_{SRD,r}^{3}}}} \qquad (13)$$

here, $r_{SRD,2}$：outside radius of damper rubber ［m］, $r_{SRD,1}$：inside radius of damper rubber ［m］, l_{SRD}：distance between adhesion sides of damper rubber ［m］, l'_{SRD}：axial direction length of rubber ［m］, μ_{SRD}：radius ratio $\left(=\frac{r_{SRD,2}}{r_{SRD,1}}\right)$ ［-］.

Figure 12 indicates the values of a to these strain conversion factors. The values of the a_{SRD} decrease with the increase of the strain conversion factor greatly. This means decrease of the torsional stiffness with increase of strain occurring by rubber shape. However, the values of dynamic characteristics receiving amplitude dependency or frequency dependency respectively, cannot be evaluated as the values of these dynamic characteristics are numerical calculated based on an engine wearing experiment result. It is the reason that the relation of the exciting amplitude and forced frequency cannot be found in the torsional vibration of the engine crankshaft as shown in Figure 1. Therefore an exciting vibration machine was manufactured in order to analyze frequency dependence and the amplitude dependence influencing on the dynamic characteristics.

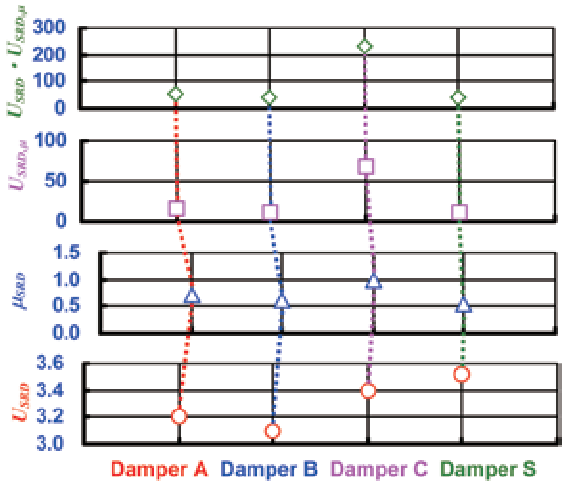

Figure 12　Rubber Shape Specifications of Test Rubber Damper

4. Exciting Torsional Vibration Experiment

The exciting torsional vibration experiment changing some exciting vibration amplitude is carried out to analyze the amplitude dependence of the dynamic characteristics.

4.1　Constitution of the Torsional Vibration Experiment Apparatus

An original exciting torsional vibration experiment apparatus was designed and manufactured to achieve the purpose.

This exciting torsional vibration experiment apparatus consists of a pulley for exciting torsional vibration, a motor, and a rubber damper wearing pulley. The shape of a pulley for exciting vibration is hexagon to generate the sixth vibration same as the test diesel engine.

An outline of the exciting torsional vibration experiment apparatus is shown in **Figure 13**. The up-and-down motion of a transmission belt occurring by an exciting torsional vibration pulley transmits the sixth vibration to a rubber damper wearing pulley. The shape change of the exciting torsional vibration pulley can change the amplitude of the sixth vibration.

4.2　Principle of Exciting Torsional Vibration Experiment Apparatus

It is supposed that Point A_{EX} in the circumference of the

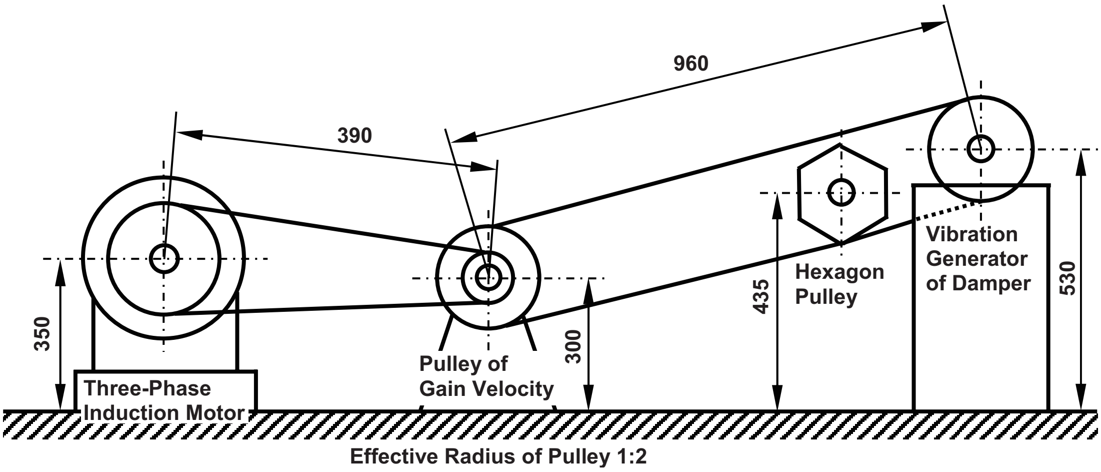

Figure 13 Schematic Diagram of the Exciting Torsional Vibration Experiment Apparatus

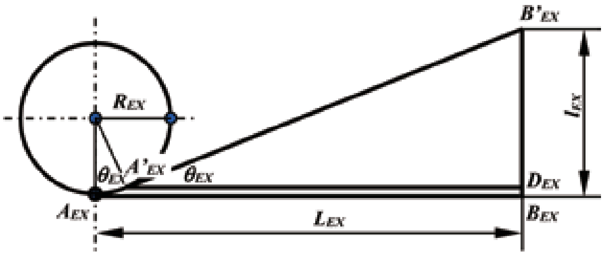

Figure 14 Principle of Exciting Torsional Vibration Experiment Apparatus

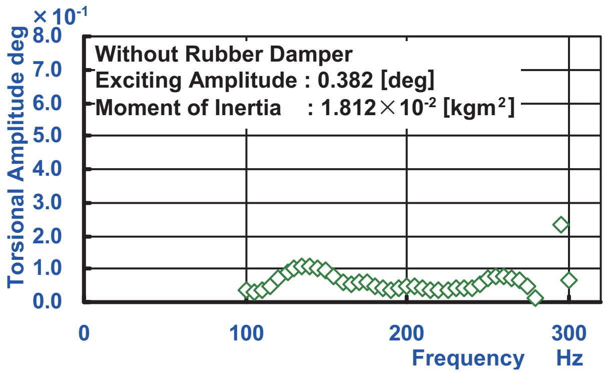

Figure 15 Exciting Torsional Vibration Amplitude Curves without a Rubber Damper [Exciting Amplitude : 0.382 [degree], Forced Frequency : 280 [Hz]]

pulley for damper wearing moves to A'_{EX}, when point B_{EX} on the transmission belt moves up to B'_{EX} by pulley for exciting torsional vibration placed between a motor and pulley for damper wearing as shown in **Figure 14**. The movement can give the rotation angle : θ_{EX} to rubber damper wearing pulley. This angle of rotation produces torsional vibration to the damper.

The relationship between torsional angular displacement : θ_{EX} and quantity of lift of the belt : l_{EX}, which is occurred by exciting torsional vibration pulley, are expressed in the next equation.

$$\sin\theta_{EX} = \frac{l_{EX} - \left(R_{EX} - R_{EX}\cdot\cos\theta_{EX}\right)}{L_{EX}} \tag{14}$$

The upper equation is arranged in θ_{EX}.

$$\theta_{EX} = \sin^{-1}_{EX}\cdot\frac{l - R_{EX}}{\sqrt{l_{EX}^2 + R_{EX}^2}} + \cos^{-1}_{EX}\cdot\frac{L_{EX}}{\sqrt{l_{EX}^2 + R_{EX}^2}} \tag{15}$$

Here, θ_{EX} : rotational angle of the pulley for wearing rubber damper [rad], R_{EX} : radius of the pulley for wearing rubber damper [m], L_{EX} : belt length between the pulley for wearing rubber damper and pulley for exciting torsional vibration amplitude [m], l_{EX} : lifted quantity of belt [m].

The quantity of lift of the belt was decided based on the torsional vibration amplitude which the experiment engine produced. As the values to substitute for equation (15), it is assumed that R_{EX}=80 [mm], L_{EX}=300 [mm] and l_{EX}= 1, 2 and 3 [mm]. As numerical calculation results, theoretical torsional amplitude θ_{EX} are 0.191, 0.382 and 0.570 [degrees].

4.3 Experimental Results

Some examples of the results of the exciting torsional vibration experiment are shown in **Figures 15** to **16**. Figure 15 shows the exciting torsional vibration amplitude curves without a rubber damper. When forced frequency exceeds 280 [Hz], it is found that the change of the exciting torsional vibration amplitude is bigger. Accordingly the frequencies range more than 280 [Hz] should be excluded from the measurement range.

Figures 17 and **18** indicate torsional vibration amplitude curves of rubber damper B and damper STD (S), but the difference of the resonance frequency can be confirmed. As the inertia moments of the inertia ring of both the rubber

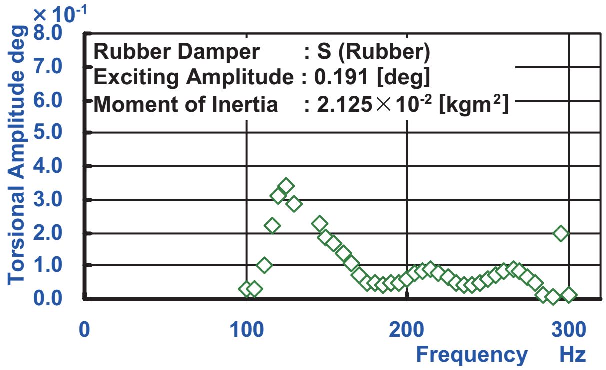

Figure 16　Torsional Vibration Amplitude Curves of Rubber Damper STD (S) [Exciting Amplitude : 0.191 [degree], Moment of Inertia : 2.125×10^{-2} [kgm^2], Forced Frequency : 280 [Hz]]

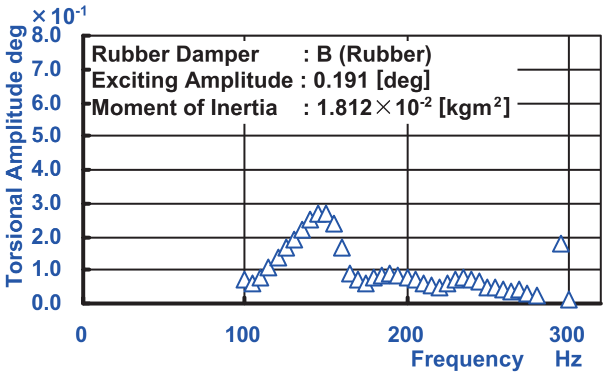

Figure 17　Torsional Vibration Amplitude Curves of Rubber Damper B [Exciting Amplitude : 0.191 [degree], Moment of Inertia : 1.812×10^{-2} [kgm^2]]

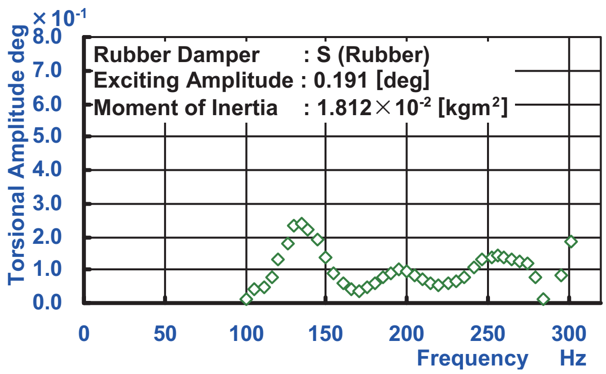

Figure 18　Torsional Vibration Amplitude Curves of Rubber Damper STD (S) [Exciting Amplitude : 0.191 [degree], Moment of Inertia : 1.812×10^{-2} [kgm^2]]

dampers are constant value, it can be supposed that the difference of the torsional stiffness is the factor of this difference. Based on this experiment result, the relations among the change of the dynamic characteristics by the exciting vibration amplitude and the rubber shape must be investigated. The dynamic characteristics values can be

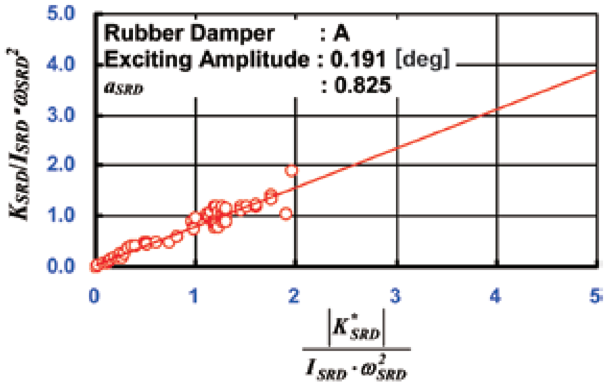

Figure 19　Relationship between Torsional Stiffness and Absolute Value of Complex Torsional Stiffness [Rubber Damper A, Exciting Amplitude : 0.191 [degree], a_{SRD} : 0.825]

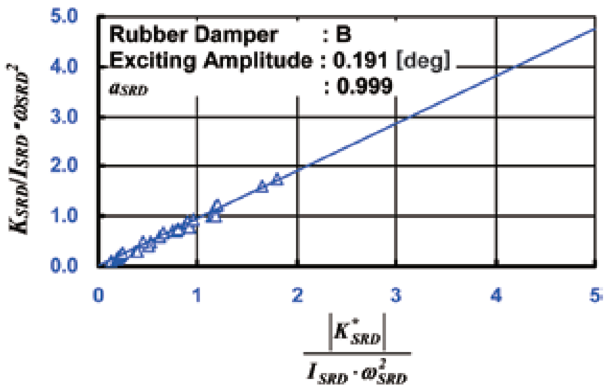

Figure 20　Relationship between Torsional Stiffness and Absolute Value of Complex Torsional Stiffness [Rubber Damper B, Exciting Amplitude : 0.191 [degree], a_{SRD} : 0.999]

numerical calculated in the above-mentioned procedure by the use of the experiment results obtained from the exciting torsional vibration experiments.

In other words it can bring each dynamic characteristics value to substitute the experiment results for equation (5) from equation (2).

4.4　Dynamic Characteristics

Figures 19 to **22** show the relations between the absolute value of the torsional stiffness $\left|K^*_{SRD}\right|$ and the torsional stiffness of each test rubber damper. These figures show that the tendencies of dynamic characteristics provided from the exciting torsional vibration experiments are the same as the tendencies of dynamic characteristics provided from the engine wearing experiments. **Figure 23** shows the relation among the ratio of torsional stiffness a_{SRD} and the exciting torsional vibration amplitude. The amplitude dependence of the dynamic characteristics of rubber damper B is the smallest and damper STD (S), is the biggest. It is supposed

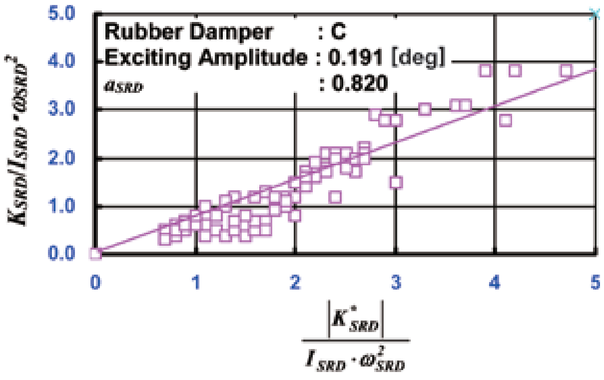

Figure 21 Relationship between Torsional Stiffness and Absolute Value of Complex Torsional Stiffness [Rubber Damper C, Exciting Amplitude : 0.191 [degree], a_{SRD} : 0.820]

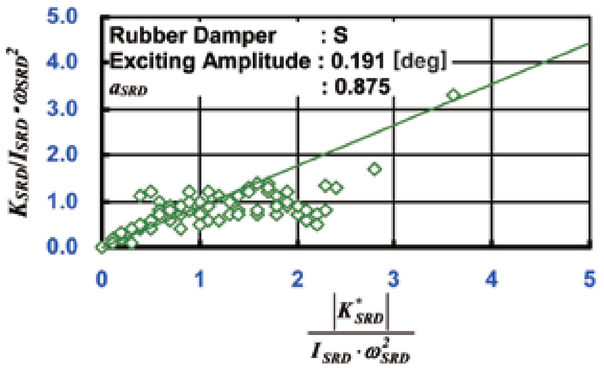

Figure 22 Relationship between Torsional Rigidity and Absolute Value of Complex Torsional Rigidity [Rubber Damper STD (S), Exciting Amplitude : 0.191 [degree], a_{SRD} : 0.875]

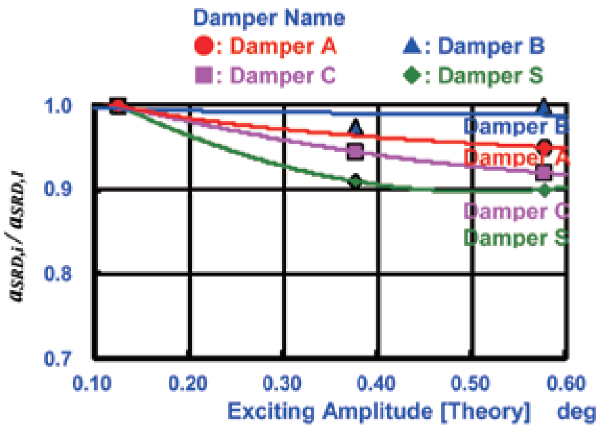

Figure 23 Relationship among the Ratio of Torsional Stiffness a_{SRD} and the Exciting Torsional Vibration Amplitude

that this root is the difference of strain produced by rubber shape. On the other hand, the difference of the value of a_{SRD} under the same exciting amplitude is considered with the difference of the area ratio as shown in **Figure 24**. This figure shows the ratio of a_{SRD} in the other amplitudes to a_{SRD}

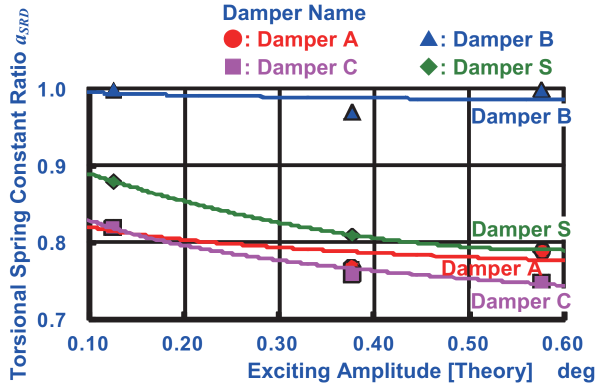

Figure 24 Relationship among the Ratio of Torsional Rigidity a_{SRD} and Shape Factor [if $a_{SRD}=1$ at Theory Exciting Torsional Amplitude = 0.191 [degree]]

in the exciting torsional amplitude 0.191 [degree].

It is clear that the dynamic characteristic values are influenced by rubber shape greatly in this way.

5. Relationship between Dynamic Characteristics and Rubber Shape

Furthermore, The relation among damping coefficient and dynamic torsional stiffness are investigated introducing the loss factor that is determined as ration of damping torque $C_{SRD}\cdot\omega_{SRD}$ to torsional stiffness K_{SRD}.

Ratio of the torsional stiffness a_{SRD} to the loss factor $\frac{C_{SRD}\cdot\omega_{SRD}}{K_{SRD}}$ are expressed in the next equation using equation (4).

$$a_{SRD}=\frac{K_{SRD}}{|K^*_{SRD}|}=\frac{K_{SRD}}{\sqrt{K^2_{SRD}+(C_{SRD}\cdot\omega_{SRD})^2}}=\frac{1}{\sqrt{1+\left(\frac{C_{SRD}\cdot\omega_{SRD}}{K_{SRD}}\right)^2}} \tag{16}$$

The upper equation can be rewritten on the $\frac{C_{SRD}\cdot\omega_{SRD}}{K_{SRD}}$

$$\left(\frac{C_{SRD}\cdot\omega_{SRD}}{K_{SRD}}\right)=\sqrt{\frac{1}{a^2_{SRD}}-1} \tag{17}$$

The loss factor is also influenced by strain.

Torsional stiffness and damping coefficient can be expressed in the next expression in consideration of rubber shape.

$$K_{SRD}=G_{SRD}\cdot U_{SRD,d}$$
$$C_{SRD}=\eta_{SRD}\cdot U_{SRD,d} \tag{18}$$

Here, G_{SRD} : young modulus [Pa], η_{SRD} : coefficient of viscosity [Nms/rad], $U_{SRD,d}$: shape factor of damper determined as ratio of second section pole moment to distance between the adhesion side [−]

As shown in expression (17), it may be said that the loss coefficient is in proportion to strain as it changes with the values of the a_{SRD} as follows.

$$\left(\frac{C_{SRD}\cdot\omega_{SRD}}{K_{SRD}}\right)=\left(\frac{\mu_{SRD}\cdot\omega_{SRD}}{G_{SRD}}\right)\propto\gamma_{SRD}=U_{SRD,\mu}\cdot\theta_{SRD,Ro} \quad (19)$$

Figure 25 shows the relation among the loss factor and the a_{SRD} to $\frac{\mu_{SRD}\cdot\omega_{SRD}}{G_{SRD}}$. Each curve is drawn based on equations (17) and (18). The dynamic characteristics provided by the experiment results are expressed on each curve. damper B is smallest, and damper STD (S), is biggest in the change of each dynamic characteristic value with increase of the exciting torsional vibration amplitude. In addition, damper B is maximum in a value of a_{SRD}, and damper C becomes the minimum of the test dampers. It is proper to think that these differences under the same exciting torsional vibration condition are caused by rubber shape. Area ratio affects the change of the a_{SRD} in the exciting vibration amplitude mainly, and the values of the a_{SRD} are influenced in strain conversion factor.

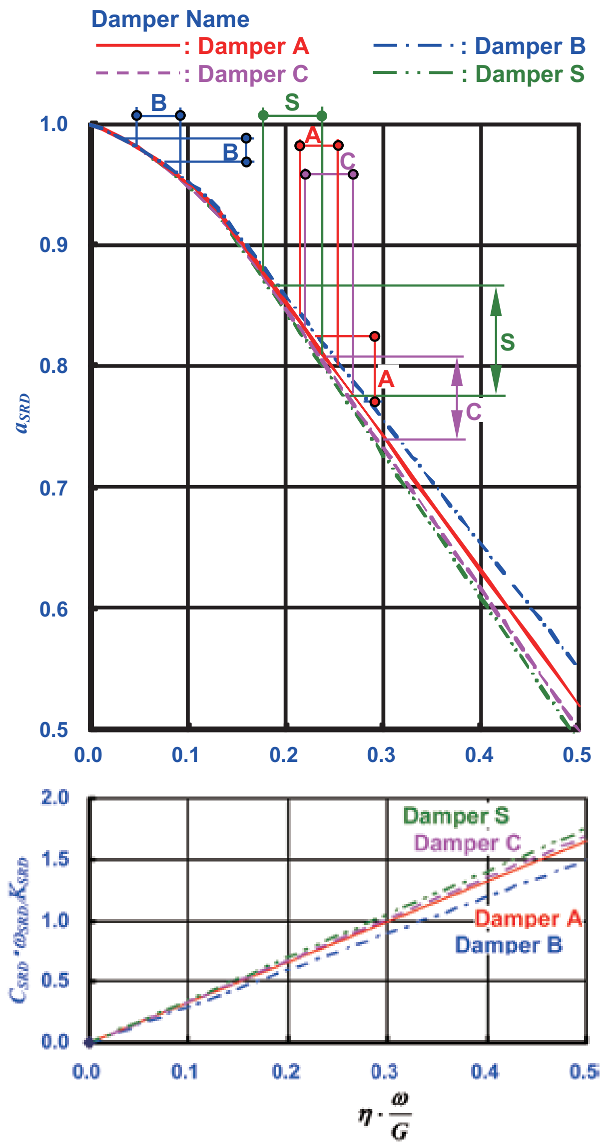

Figure 25 Relationship among a_{SRD} and Loss Factor

Equation (4) to numerical calculate absolute value of the torsional stiffness $\left|K^*_{SRD}\right|$ can be rewritten in the next equation.

$$\left|K^*_{SRD}\right|\sqrt{K^2_{SRD}+\left(C_{SRD}\cdot\omega_{SRD}\right)^2}=\frac{I_{SRD}\cdot\omega^2_{SRD}}{M_{SRD,3}} \quad (20)$$

here, $M_{SRD,3}$: ratio of amplitude of rubber to amplitude of inertia ring. [−]

In other words the first clause $I_{SRD}\cdot\omega^2_{SRD}$ of the most right side of the upper equation is a clause without the influence of the rubber shape. In contrast, the second clause $\frac{a_{SRD}}{M_{SRD,3}}$ is a clause with the influence of the rubber shape. Figure 25 shows $\frac{a_{SRD}}{M_{SRD,3}}$ in the change of exciting torsional vibration amplitudes obtained from the results of engine wearing experiments and torsional exciting vibration experiments. This figure indicates that $\frac{a_{SRD}}{M_{SRD,3}}$ of all of the test dampers decrease with increase of exciting torsional amplitudes. The tendency to decrease is strong in a range of the smaller exciting torsional vibration amplitude, and a decreasing tendency to decrease becomes in particular weak in a range of the bigger exciting torsional vibration amplitude. Figure 26 indicates the changes of $\frac{a_{SRD}}{M_{SRD,3}}$ in the exciting amplitudes of test damper obtained from the engine wearing experiments and the exciting vibration experiments.

It is estimated that strains occurring in the rubber of each test damper are different by rubber shape in a constant of the exciting torsional vibration amplitude. If the relation of exciting vibration torque T_{EX} and torsional stiffness K_{SRD} are $T_{SRD}=K_{SRD}\cdot\theta_{SRD,Ro}$, the substitution of equation (8) or (12) into this equation give the next equation.

$$T_{EX}=K_{SRD}\cdot\theta_{SRD,Ro}=K_{SRD}\cdot\frac{\gamma_{SRD}}{U_{SRD,\mu}} \quad (21)$$

As T_{EX} and K_{SRD} of all test dampers are approximately equal under the same exciting torsional vibration amplitude condition as shown in Figure 26, $\frac{\gamma_{SRD}}{U_{SRD,\mu}}$ is also nearly equal. However, the quantity of strain is different every damper greatly because the strain γ_{SRD} is in proportion to the strain conversion factor $U_{SRD,\mu}$. Accordingly this fact means that a damper with bigger quantity of strain has bigger

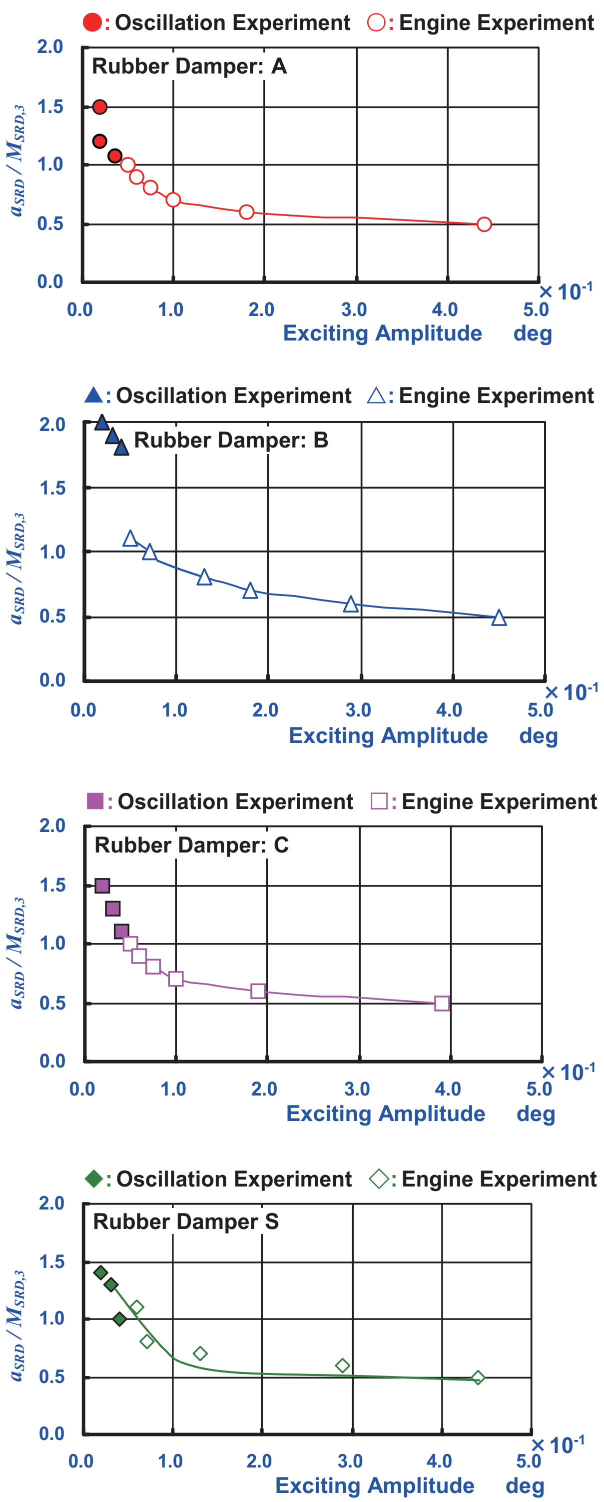

Figure 26 The Change of Exciting Torsional Vibration Amplitudes obtained from the Results of Engine Wearing Experiments and Exciting Torsional Vibration Experiments

damping coefficient. Such a damper causes increase of the vibration absorption energy and will show larger temperature dependence.

The main requirement item to a rubber damper is reduction of the bigger amplitude of the crankshaft system. Rubber damper used natural rubber is expected a control on vibration effect by the torsional stiffness more than damping coefficient. Furthermore, a control on vibration effect in the bigger exciting torsional vibration amplitude is important. These mean that damper B with the smallest shape factor and strain conversion factor of the test dampers has the biggest control on vibration effect.

6. Conclusions

This study refers to the investigation on the relation between dynamic characteristics and rubber shape through engine wearing experiment and exciting torsional vibration experiment using four kinds of rubber dampers with only different rubber shape. As some results of this investigation, the knowledge is provided as follows.

[1] The dynamic characteristics of the damper depend on shape factor.

[2] The dependent tendency presents conspicuously with increase of the shape factor.

[3] Damper with smaller shape factor contributes to easily predict dynamic characteristics on the design stage. However, the rubber damper of the small shape factor is hard to satisfy a torsional stiffness to tune to the vibration mode of the crankshaft system of the engine.

Definition of Symbols

Show in major symbols and definitions.

a_{SRD} : Ratio of torsional stiffness [−].

C_{SRD} : Damping coefficient of damper rubber [Nms/rad].

G_{SRD} : Young modulus [Pa].

I_{SRD} : Inertia moment of damper inertia ring [kgm^2].

K_{SRD} : Torsional rigidity of damper rubber [Nm/rad].

$\left|K^*_{SRD}\right|$: Absolute value of the torsional stiffness [Nm/rad].

L : Belt length between the pulley for wearing rubber damper and pulley for exciting vibration amplitude [m].

L_{EX} : Belt length between the pulley for wearing rubber damper and pulley for exciting vibration amplitude [m].

l_{EX} : Lifted quantity of belt [m].

l_{SRD} : Distance between adhesion sides of damper rubber [m].

l'_{SRD} : Axial direction length of rubber [m].

M_{SRD} : Amplitude ratio $\left(=\dfrac{\theta_{SRD,do}}{\theta_{SRD,po}}\right)$ [-].

$M_{SRD,3}$: Ratio of amplitude of rubber to amplitude of inertia ring. [−].

$\boldsymbol{R}$：Radius of the pulley for wearing rubber damper［m］.

$\boldsymbol{R_{EX}}$：Radius of the pulley for wearing rubber damper［m］.

$\boldsymbol{r_{SRD,1}}$：Inside radius of damper rubber［m］.

$\boldsymbol{r_{SRD,2}}$：Outside radius of damper rubber［m］.

$\boldsymbol{T_{EX}}$：Exciting torsional vibration torque［Nm］.

$\boldsymbol{U_{SRD,a}}$：Area ratio of axial direction adhesion type ［−］.

$\boldsymbol{U_{SRD,d}}$：Shape factor of damper determined as ratio of second section pole moment to distance between the adhesion side［−］.

$\boldsymbol{U_{SRD,r}}$：Area ratio of radial direction adhesion type［−］.

$\boldsymbol{U_{SRD,a,\mu}}$：Strain conversion factor of axial direction adhesion type［−］.

$\boldsymbol{U_{SRD,r,\mu}}$：Strain conversion factor of radial direction adhesion type［−］.

$$\gamma_{SRD,a}=\frac{1}{1-\mu_{SRDF,a}\cdot\sqrt{\dfrac{3\cdot(1-\mu_{SRD})}{1-\mu_{SRD,a}^{3}}}}\cdot\theta_{SRD,a,Ro}=$$

$\boldsymbol{U_{SRD,a,\mu}\cdot\theta_{SRD,a,Ro}}$：Strain of axial direction adhesion type.［−］.

$$\gamma_{SRD,r}=\frac{3\cdot U_{SRD,r}\cdot(1+\mu_{SRD,r})\cdot(1+\mu_{SRD,r}^{2})}{2\cdot(1-\mu_{SRD,r}^{3})}\cdot\theta_{SRD,r,Ro}=$$

$\boldsymbol{U_{SRD,r,\mu}\cdot\theta_{SRD,r,Ro}}$：Strain of radial direction adhesion type［−］.

$\boldsymbol{\eta_{SRD}}$：Coefficient of viscosity［Nms/rad］.

$\boldsymbol{\mu_{SRD}}$：Radius ratio $\left(=\dfrac{r_{SRD,2}}{r_{SRD,1}}\right)$［−］.

$\boldsymbol{\theta_{EX}}$：Rotational angle of the pulley for wearing rubber damper［rad］.

$\boldsymbol{\theta_{SRD,d}}$：Torsional vibration angular displacement of damper housing［rad］.

$\boldsymbol{\theta_{SRD,do}}$：Torsional vibration amplitude of damper housing［rad］.

$\boldsymbol{\theta_{SRD,p}}$：Torsional vibration angular displacement of the damper inertia ring［rad］.

$\boldsymbol{\theta_{SRD,po}}$：Torsional vibration amplitude of the damper inertia ring［rad］.

$$\rho_{SRD,a}=r_{SRD,a,2}\cdot\sqrt{\frac{(1-\mu_{SRD,a}^{3})}{3\cdot(1-\mu_{SRD,a})}}$$

：Representative position of the strain of axial direction adhesion type［−］.

$$\rho_{SRD,r}=\frac{3\cdot r_{SRD,r,2}\cdot(1+\mu_{SRD,r}^{4})}{4\cdot(1-\mu_{SRD,r}^{3})}\cdot\theta_{SRD,r,Ro}$$

：Representative position of the strain of radial direction adhesion type［−］.

$\boldsymbol{\omega_{SRD}}$：Forced frequency［rad/s］.

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